

COMBINED MATHS 2024 - I

① $\sum_{r=1}^{n+1} (2r-1) = (n+1)^2$

$n=1$ ට $LHS = \sum_{r=1}^2 (2r-1) = 2(1)-1 + 2(2)-1 = 4$
 $n=1$ ට $RHS = (1+1)^2 = 4$
 $\therefore n=1$ ට ඉහත සමාන වේ. (5)

$n = p$, $p \in \mathbb{Z}^+$ සඳහා ඉහත සමාන වේ යැයි උපකල්පනය කරමු.

$\sum_{r=1}^{p+1} (2r-1) = (p+1)^2$ (5)

$n = p+1$ ට $\sum_{r=1}^{p+2} (2r-1) = \sum_{r=1}^{p+1} (2r-1) + 2(p+2)-1 = (p+1)^2 + 2p+3$
 $= p^2 + 2p+1 + 2p+3 = p^2 + 4p+4 = (p+2)^2$
 $\sum_{r=1}^{p+2} 2r-1 = (p+2)^2$ (5)

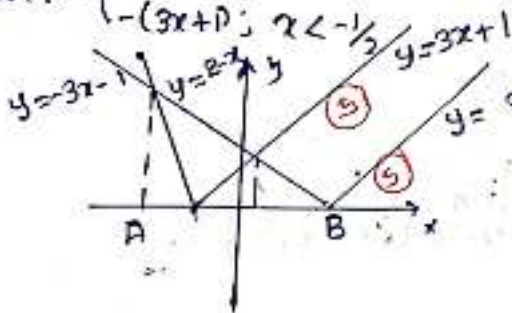
$\therefore n = p+1$ ටද ඉහත සමාන වේ. එබැවින් ඉහත ප්‍රකාරයෙන් $n \in \mathbb{Z}^+$ සඳහා ඉහත සමාන වේ. (5)

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2) $\left| \frac{3x+1}{x-2} \right| > 1$

$|x-2| = \begin{cases} x-2 & x \geq 2 \\ -(x-2) & x < 2 \end{cases}$

$3x+1 = \begin{cases} 3x+1 & x > -1/3 \\ -(3x+1) & x < -1/3 \end{cases}$



$\begin{cases} y = -3x-1 \\ y = 2-x \end{cases} \Rightarrow x = -3/2$ (A) (5)

$\begin{cases} y = 3x+1 \\ y = 2-x \end{cases} \Rightarrow x = 1/2$ (B) (5)

\therefore අගය වන්නේ $x < -3/2$ හෝ $1/2 < x < 2$ හෝ $x > 2$ (5)

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3) $T_{n+1} = n C_r (px)^r$
 $= n C_r p^r x^r$

$r=1$ ට

$n C_1 p = 8$

$np = 8$ (1)

$r=2$ ට

$n C_2 p^2 = 24$

$\frac{n! p^2}{2!(n-2)!} = 24$ (5)

$n \frac{(n-1)}{2} p^2 = 24$ (2)

① හා ② ට

$\frac{n(n-1) \cdot 64}{2} = 24$ (5)

$4(n-1) = 3n$
 $n = 4$ (5)

$p = 5$ (5)

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$$\textcircled{4} (2+i)^3 = 2^3 + 3 \cdot 2^2 i + 3 \cdot (2) i^2 + i^3$$

$$= 8 + 12i - 6 - i$$

$$= 2 + 11i \textcircled{5}$$

$$\therefore b = 11 \textcircled{5}$$

$(2+i)$, $z^3 + pz + q = 0$ को गुणकारी गर

$$(2+i)^3 + p(2+i) + q = 0 \textcircled{5}$$

$$(2+11i) + p(2+i) + q = 0$$

$$(2+2p+q) + (11+p)i = 0$$

$$\text{Im}(z_0) = 0$$

$$\text{Re}(z_0) = 0$$

$$11+p = 0$$

$$2+2p+p = 0$$

$$p = -11 \textcircled{5}$$

$$2+2(-11)+q = 0$$

$$q = 20 \textcircled{5}$$

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$$\textcircled{5} \lim_{x \rightarrow 0} \frac{(\sqrt{x+2} - \sqrt{2})(1 + x \sin 3x - \cos 2x) \times (\sqrt{x+2} + \sqrt{2})}{x^2 \sin x (\sqrt{x+2} + \sqrt{2})} \textcircled{5}$$

$$\lim_{x \rightarrow 0} \frac{x(1 + x \sin 3x - \cos 2x)}{x^2 \sin x (\sqrt{x+2} + \sqrt{2})} = \frac{x(x \sin 3x + 2 \sin^2 x)}{x^2 \sin x (\sqrt{x+2} + \sqrt{2})}$$

$$\lim_{x \rightarrow 0} \left[\frac{\sin 3x}{\sin x} + \frac{2 \sin^2 x}{x} \right] \frac{1}{\sqrt{x+2} + \sqrt{2}}$$

$$3 \lim_{x \rightarrow 0} \left[\frac{\sin 3x}{3x} + 2 \lim_{x \rightarrow 0} \frac{\sin x}{x} \right] \lim_{x \rightarrow 0} \left[\frac{1}{\sqrt{x+2} + \sqrt{2}} \right] \textcircled{5}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x}$$

$$\frac{3+2}{2\sqrt{2}} = \frac{5\sqrt{2}}{4} \textcircled{5}$$

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$$06) y = \tan^{-1} \sqrt{3x}$$

$$\frac{dy}{dx} = \frac{1}{1+3x^2} \times \sqrt{3} \times \frac{1}{2} x^{-1/2} \quad (5)$$

$$\frac{2}{\sqrt{3}} dy = \frac{dx}{(1+3x)\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{\sqrt{3}}{2(1+3x^2)\sqrt{x}} \quad (5)$$

$$\frac{2}{\sqrt{3}} \int dy = \int \frac{dx}{(1+3x)\sqrt{x}}$$

$$\int \frac{dx}{\frac{1}{3}(1+3x)\sqrt{x}} = \frac{2}{\sqrt{3}} [\tan^{-1} \sqrt{3x}]_{\frac{1}{3}}^1 \quad (5)$$

$$\int \frac{dx}{\frac{1}{3}(1+3x)\sqrt{x}} = \frac{2}{\sqrt{3}} [\tan^{-1} \sqrt{3} - \tan^{-1}(1)] \quad (5)$$

$$= \frac{2\sqrt{3}}{3} \left[\frac{\pi}{3} - \frac{\pi}{4} \right]$$

$$= \frac{\sqrt{3}\pi}{18} \therefore n=18 \quad (5)$$

[25]

$$\Rightarrow x = 2(1+\sin\theta) \quad y = 2\cos 2\theta$$

$$\frac{dx}{d\theta} = 2\cos\theta \quad \frac{dy}{d\theta} = -4\sin\theta \quad (5)$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} = \frac{-4\sin 2\theta}{2\cos\theta} = \frac{-4 \times 2\sin\theta\cos\theta}{2\cos\theta} = -4\sin\theta \quad (5)$$

Equation of the normal

$$y - 2\cos 2\theta = -4\sin\theta (x - 2(1+\sin\theta)) \quad (5)$$

$$y + 4\sin\theta x - 2\cos 2\theta - 8\sin\theta(1+\sin\theta) = 0$$

$$y + 4\sin\theta x - 8\sin\theta - 8\sin^2\theta - 2\cos 2\theta = 0$$

$$y + 4\sin\theta x - 8\sin\theta - 8\sin^2\theta - 2(1-2\sin^2\theta) = 0 \quad (5)$$

$$y + 4\sin\theta x - 8\sin\theta - 4\sin^2\theta - 2 = 0$$

$$y + 4\sin\theta x - 2(2\sin\theta + 4\sin\theta + 1) = 0 \quad (5)$$

[25]

$$8) \frac{y-2}{x-1} = 1 = t \Rightarrow \frac{y-2}{x-1} = t \quad \text{Let } t = \frac{y-2}{x-1}$$

$$x = 1+t, \quad y = 2+t \quad (5)$$

$$(+)\ 7t+1=15$$

$$(-)\ 7t+1=-15$$

$$t=2$$

$$7t=-16$$

$$t = -\frac{16}{7}$$

$$| \frac{3(1+t)+4(2+t)-10}{\sqrt{3^2+4^2}} | = 3 \quad (5)$$

$$| \frac{7t+1}{5} | = 3 \therefore P \equiv (3, 4) \text{ or } P = \left(-\frac{9}{7}, -\frac{2}{7} \right) \quad (5)$$

$$7t+1 = \pm 15 \quad (5)$$

[25]

09) $g=0$ හි මධ්‍යස්ථය C වන

$$2g = -4, 2f = 6$$

$$g = -2, f = 3$$

$$C = (-g, -f) = (2, -3)$$

$$S=0 \text{ හි අඩංගු } P = \sqrt{g^2 + f^2 - c}$$

$$P = \sqrt{(-2)^2 + 3^2 - (-12)}$$

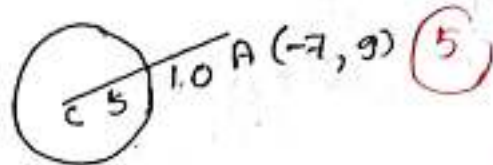
$$P = 5 \quad (5)$$

$$AC \text{ දිග} = \sqrt{(-7-2)^2 + (9-3)^2}$$

$$= 15 \quad (5)$$

AC හි මධ්‍යස්ථය අඩංගු

A අඩංගු මධ්‍යස්ථය සිටින්නේ පිහිටිය



$$CP : PA = 5 : 10 = 1 : 2 \quad (5)$$

$$P = \frac{1 \times (-7) + 2 \times 5}{1+2}, \frac{1 \times 9 + 2 \times (-3)}{1+3}$$

$$P = (-1, 1) \quad (5)$$

25

$$10) \cos 52^\circ + \cos 68^\circ + \cos 172^\circ$$

$$= 2 \cos 60^\circ (\cos 8^\circ + \cos 172^\circ) \quad (5)$$

$$= 2 \times \frac{1}{2} (\cos 8^\circ + \cos 172^\circ) \quad (5)$$

$$= \cos 8^\circ + \cos (180 - 172) \quad (5)$$

$$= \cos 8^\circ - \cos 8^\circ \quad (5)$$

$$= 0 \quad (5)$$

25

B මොනවද

$$\Phi 1] a) f(x) = x^2 + kx - 5 = 0 \quad \alpha \quad \beta$$

$k \in \mathbb{R}$

$$\alpha + \beta = -k \quad (5) \quad \alpha\beta = -5 \quad (5)$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (-k)^2 - 2(-5) \quad (5)$$

$$= k^2 + 10 \quad (5)$$

$$5(\alpha^2 + \beta^2) = 7\alpha^2\beta^2$$

$$5(k^2 + 10) = 7 \times 25 \quad (5)$$

$$k^2 + 10 = 35$$

$$k^2 = 25 \quad (5)$$

$$k = \pm 5$$

$$\alpha^2\beta^2 = (\alpha\beta)^2$$

$$= (-5)^2 \quad (5)$$

$$= 25 \quad (5)$$

$$k = 5 \text{ හෝ } k = -5$$

$$(5)$$

$$(5)$$

$$k = 5 \text{ හෝ } \alpha^2 + \beta^2 = 35 \quad \text{මුළු ගුණිතය} \quad \frac{1}{\alpha^2} \times \frac{1}{\beta^2} = \frac{1}{\alpha^2\beta^2} = \frac{1}{25} \quad (5)$$

මුළු වෙනස $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2} = \frac{35}{25} = \frac{7}{5}$ (5)

$\frac{1}{\alpha^2}$ හෝ $\frac{1}{\beta^2}$ මුළු වන ස්වභාවය $x^2 - [\frac{1}{\alpha^2} + \frac{1}{\beta^2}]x + \frac{1}{\alpha^2} \frac{1}{\beta^2} = 0$

$x^2 - (\frac{7}{5})x + \frac{1}{25} = 0$ (5)

$25x^2 - 35x + 1 = 0$ (5)

$y = \begin{cases} \frac{\alpha^3 \beta + 1}{\alpha^2} = \alpha \beta + \frac{1}{\alpha^2} = -5 + x \\ \frac{\alpha \beta^3 + 1}{\beta^2} = \alpha \beta + \frac{1}{\beta^2} = -5 + x \end{cases}$

$y = -5 + x \Rightarrow x = (y + 5)$

$25(y + 5)^2 - 35(y + 5) + 1 = 0$ (5)

$25y^2 + 250y + 625 - 35y - 175 + 1 = 0$

$25y^2 + 215y + 451 = 0$ (5)

$y \rightarrow x$

$25x^2 + 215x + 451 = 0$

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b) $f(x)$ බහුපදය $(x-a)$ හි බෙදුමේ ඍණය $f(a)$ වේ. (5)

$f(x) = (x-a)g(x) + R$ (5)

$x=a$ විට $f(a) = R$ (5)

$f(x) = (3x^2 + x - 2)h(x) + x + 2$

$f(x) = (x+1)(3x-2)h(x) + x + 2$ — (1) (5)

$g(x) = (x-1)k(x) + x$

$g(x) = (x-1)(x+1)k(x) + x$ — (2) (5)

(1) + (2) $f(x) + g(x) = (x+1)(3x-2)h(x) + x + 2 + (x-1)(x+1)k(x) + x$ (5)

$= (x+1) [(3x-2)h(x) + (x-1)k(x) + 2]$ (5)

$\therefore (x+1)$ යනු $f(x) + g(x)$ හි නියමකරු. (5)

ඍණ ඉරේසියට අනුව

$f(x) \cdot g(x)$ බහුපදය $(x+1)$ හි බෙදුමේ ඍණය $f(-1) \cdot g(-1)$ වේ. (5)

(1) හි $f(-1) = 1$ (5)

ඍණය $f(-1) \cdot g(-1) = 1 \times (-1) = -1$ (5)

(2) හි $g(-1) = -1$ (5)

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12) a) (1)

භින්නකය (4)	එකතුව (9)	වර්ගකය (2)	ආනත ${}^4C_2 \times {}^9C_6 \times 2C_2 = 54$
2	8	2	${}^4C_3 \times {}^9C_7 \times 2C_2 = 144$
3	7	2	${}^4C_4 \times {}^9C_6 \times 2C_2 = 84$
4	6	2	
			ආනත = 282

ii) ආනත

$3! [4! \times 6! \times 2!]$

$6 [24 \times 720 \times 2]$

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iii) මූල ආනත

$12!$

භින්නකය වර්ගකය ගණ වෙනස්කිරීම $9! \times 4!$

භින්නකය වර්ගකය ගණ වෙනස්

රහස්කිරීම $12! - 9! \times 4!$

10

75

b) $U_r = \frac{r \cdot 2^r}{(r+2)!}$

$U_r = \frac{A \cdot 2^r}{(r+1)!} + \frac{B \cdot 2^r}{(r+2)!}$

$\frac{r \cdot 2^r}{(r+2)!} = \frac{A \cdot 2^r}{(r+1)!} + \frac{B \cdot 2^r}{(r+2)!}$

$r = A(r+2) + B$

අනුකූලව සවන කර

$r \rightarrow A = 1$

$r=0 \rightarrow 2A + B = 0, B = -2$

$U_r = \frac{2^r}{(r+1)!} - \frac{2 \cdot 2^r}{(r+2)!}$

$f(r) = \frac{2^r}{(r+1)!}$ නම් $f(r+1) = \frac{2^{r+1}}{(r+2)!}$ වේ.

එවිට $U_r = f(r) - f(r+1)$ වේ.

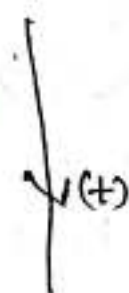
$r=1, U_1 = f(1) - f(2)$

$r=2, U_2 = f(2) - f(3)$

$r=3, U_3 = f(3) - f(4)$

$r=n-2, U_{n-2} = f(n-2) - f(n-1)$

$r=n-1, U_{n-1} = f(n-1) - f(n)$



$$x = n, \quad U_n = f(n) - f(n+1)$$

$$U_1 + U_2 + U_3 + \dots + U_n = f(1) - f(n+1)$$

$$\sum_{r=1}^n U_r = \frac{2}{2!} - \frac{2^{n+1}}{(n+2)!} \quad (10)$$

$$\sum_{r=1}^n U_r = 1 - \frac{2^{n+1}}{(n+2)!} \quad (5)$$

$$\sum_{r=1}^n W_r = 1 - \sum_{r=1}^{n-1} U_r = 1 - \left[1 - \frac{2^n}{(n+1)!} \right] \quad (10)$$

$$= \frac{2^n}{(n+1)!} \quad (5)$$

75

$$3) \quad A = \begin{pmatrix} x & -2 \\ -4 & 1 \end{pmatrix} \quad \text{Det}(A) = -2 \quad A = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}$$

$$x - (-4)(-2) = -2$$

$$x - 8 = -2$$

$$x = 6 \quad (5)$$

$$\text{LHS} = A^2 = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix} \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix} = \begin{pmatrix} 44 & -14 \\ -28 & 9 \end{pmatrix} \quad (5)$$

$$\text{RHS} = 7A + 2I = 7 \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix} + 2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 44 & -14 \\ -28 & 9 \end{pmatrix} \quad (5)$$

$$\text{LHS} = \text{RHS}$$

$$A^2 = 7A + 2I \quad (5)$$

$$I = \frac{A^2 - 7A}{2} = A \left(\frac{1}{2}A - 7I \right) \quad (5)$$

$$A^{-1} = \frac{1}{2} (A - 7I) = \frac{1}{2} \left\{ \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix} - 7 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\} \quad (5)$$

$$= \frac{1}{2} \begin{pmatrix} -1 & -2 \\ -4 & -6 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & -1 \\ -2 & -3 \end{pmatrix} \quad (5)$$

$$A \cdot B^T = I + A$$

$$A^{-1}A \cdot B^T = A^{-1}I + A^{-1}A$$

$$I \cdot B^T = A^{-1} + I$$

$$B^T = A^{-1} + I \quad (5)$$

$$B^T = \begin{bmatrix} -\frac{1}{2} & -1 \\ -2 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (5)$$

$$B^T = \begin{bmatrix} \frac{1}{2} & -1 \\ -2 & -2 \end{bmatrix} \quad (5)$$

$$B = \begin{bmatrix} \frac{1}{2} & -2 \\ -1 & -2 \end{bmatrix} \quad (5)$$

$$B C = I + B$$

$$B^{-1} C = I + A$$

$$B \cdot B^{-1} C = B I + B A$$

$$I C = B + B A \quad (5)$$

$$C = B + B A$$

$$C = \begin{bmatrix} \frac{1}{2} & -2 \\ -1 & -2 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} & -2 \\ -1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix} \quad (5)$$

$$C = \begin{bmatrix} \frac{1}{2} & -2 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 11 & -3 \\ 2 & 0 \end{bmatrix} \quad (5)$$

$$C = \begin{bmatrix} 2\frac{1}{2} & -5 \\ 1 & -2 \end{bmatrix} \quad (5)$$

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1) $Z = \cos \theta + i \sin \theta$ නම් Z ඉහත ප්‍රකාරයේ ආකාරයේ

$$Z^n = \cos n\theta + i \sin n\theta \quad (1) \quad (1) + (2)$$

$$\frac{1}{2} Z^n = \cos n\theta + i \sin(-n\theta)$$

$$Z^n + \frac{1}{2} Z^n = \cos n\theta + i \sin n\theta + \cos n\theta - i \sin n\theta \quad (5)$$

$$\frac{1}{2} Z^n = \cos n\theta - i \sin n\theta \quad (2)$$

$$Z^n + \frac{1}{2} Z^n = 2 \cos n\theta \quad (3)$$

$$(1) - (2) \quad Z^n - \frac{1}{2} Z^n = \cos n\theta + i \sin n\theta - (\cos n\theta - i \sin n\theta)$$

$$Z^n - \frac{1}{2} Z^n = 2i \sin n\theta \quad (4) \quad (5)$$

$$\frac{Z^n + \frac{1}{2} Z^n}{Z^n - \frac{1}{2} Z^n} = \frac{2 \cos n\theta}{2i \sin n\theta} \quad (5)$$

$$\frac{(Z^n)^2 - 1}{(Z^n)^2 + 1} = i \tan n\theta \Rightarrow \frac{Z^n - 1}{Z^n + 1} = i \tan n\theta \quad (5)$$

$$Z_1 = \frac{\sqrt{3} + i}{2} \text{ වෙස වෙස}$$

$$Z_2 = \frac{i - \sqrt{3}}{2} \text{ වෙස වෙස}$$

$$Z_1 = \frac{\sqrt{3}}{2} + \frac{1}{2}i = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \quad (5)$$

$$Z_2 = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$$

$$Z_1^6 = \cos \pi + i \sin \pi$$

$$Z_2 = \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \quad (5)$$

$$Z_1^6 = -1 + 0 \quad (5)$$

$$Z_2^6 = \cos 5\pi + i \sin 5\pi \quad (5)$$

$$Z_1^6 = -1$$

$$Z_2^6 = \cos \pi + i \sin \pi$$

$$(5)$$

$$= -1 + 0$$

$$Z_2^6 = -1 \quad (5)$$

$$\text{LHS } \left(\frac{\sqrt{3}+2}{2}\right)^6 + \left(\frac{1-\sqrt{3}}{2}\right)^6 = Z_1^6 + Z_2^6 = (-1) + (-1)$$

$$\text{(5)} = -2 = \text{RHS}$$

(5)

70

$$14) a) f(x) = \frac{x(x-2)}{(x+1)(x-3)}$$

$$f'(x) = \frac{(x+1)(x-3)[x+x-2] - x(x-2)[x+1+x-3]}{(x+1)^2(x-3)^2} \text{ (5)}$$

$$f'(x) = \frac{(x+1)(x-3)(2x-2) - x(x-2)(2x-2)}{(x+1)^2(x-3)^2}$$

$$= \frac{2(x-1)[x^2 - 2x - 3 - x^2 + 2x]}{(x+1)^2(x-3)^2} \text{ (5)}$$

$$= \frac{2(x-1)(-3)}{(x+1)^2(x-3)^2}$$

$$f'(x) = \frac{-6(x-1)}{(x+1)^2(x-3)^2}$$

$$f''(x) = \frac{(x+1)^2(x-3)^2[-6] + 6(x-1)[(x+1)^2 \cdot 2(x-3) + (x-3)^2 \cdot 2(x+1)]}{(x+1)^4(x-3)^4} \text{ (5)}$$

$$= \frac{+6[-(x+1)(x-3) + 2(x+1)(x-3) + 2(x-3)(x+1)]}{(x+1)^3(x-3)^3}$$

$$f''(x) = \frac{+6[3x^2 - 6x + 7]}{(x+1)^3(x-3)^3}$$

$$f''(x) = \frac{18\left[(x-1)^2 + \frac{4}{3}\right]}{(x+1)^3(x-3)^3} \quad f''(x) \neq 0$$

$\therefore f(x)$ ප්‍රභවනයට නැතිවීමට ලක්වන ලදී.
ඉහතරට ගොනෙත

$f'(x) = 0$ දී ප්‍රභවනයට හැරවුණු ලදී පවති

$$\frac{-6(x-1)}{(x+1)^2(x-3)^2} = 0$$

$$x = 1$$

$(1, \frac{1}{4})$ හැරවුණු ලදී

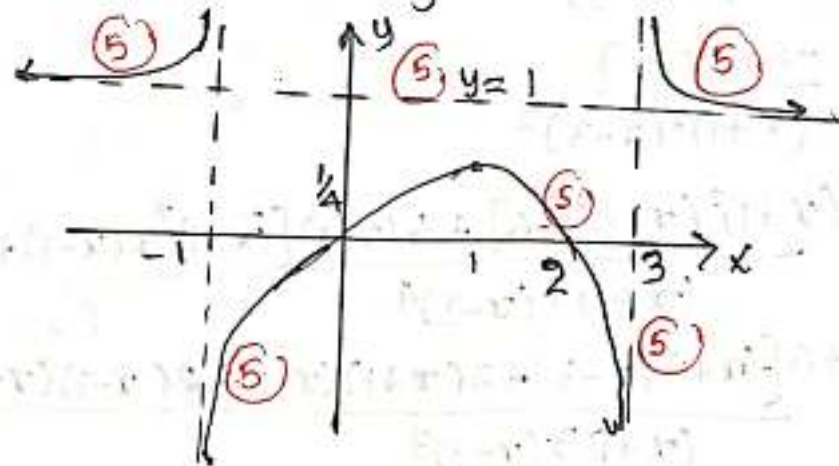
$-d < x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$	$x = 3$	$3 < x < +\infty$
$f'(x) > 0$		$f'(x) > 0$		$f'(x) < 0$		$f'(x) < 0$
↗		↘		∩		∪
(5)		(5)		(5)		(5)
അർദ്ധമാനമാ		ഉൾമാ		അർദ്ധമാനമാ		

$x \rightarrow -1^-$ യോ $y \rightarrow +\infty$ $x \rightarrow 3^-$ യോ $y \rightarrow -\infty$
 $x \rightarrow -1^+$ യോ $y \rightarrow -\infty$ $x \rightarrow 3^+$ യോ $y \rightarrow +\infty$

ഖണ്ഡ അർദ്ധമാനമാ $\rightarrow x = -1$ (5)
 $x = 3$ (5)

$$\lim_{x \rightarrow \pm\infty} \frac{x(x-2)}{(x+1)(x-3)} = \lim_{x \rightarrow \pm\infty} \frac{(1-\frac{2}{x})}{(1+\frac{1}{x})(1-\frac{3}{x})} = 1 \quad (5)$$

\therefore ഖണ്ഡ അർദ്ധമാനമാ $y = 1$



80

b) 1) $V = \frac{2}{3}\pi r^3 + \pi r^2 h$

$$= \pi r^2 \left(\frac{2r}{3} + h \right) \quad (5)$$

$$S = 2\pi r h + 2\pi r^2 + \pi r^2$$

$$S = \pi r (2h + 3r) \quad (5)$$

ii) $\theta \pi = \pi r^2 h$

$$h = \frac{\theta}{r^2} \quad (5)$$

$$S = \pi r (2h + 3r)$$

$$\pi r \left(\frac{18}{r^2} + 3r \right)$$

$$S = \frac{18\pi}{r} + 3\pi r^2 \quad (5)$$

iii) $\frac{dS}{dr} = -\frac{18\pi}{r^2} + 6\pi r \quad (5)$

$$= \frac{6\pi}{r} [r^3 - 3] \quad (5)$$

$$\frac{dS}{dr} = 0 \text{ when } r = \sqrt[3]{3} \text{ യോ } (5)$$

$$\theta = \frac{18\pi}{\sqrt[3]{3}} + 3\pi \cdot (3)^{2/3}$$

$$S = \frac{27\pi}{\sqrt[3]{3}} \text{ m}^2 \quad (5)$$

$h = (2\pi r h + \pi r^2) \times 200$
 $h = (2\pi r^2 \times 400)$

මුළු ජනිතය C ගත්

$$C = 400\pi r h + 800\pi r^2$$

$$C = 400\pi r \left(\frac{9}{r^2}\right) + 800\pi r^2 \Rightarrow C = 400\pi \left[\frac{9}{r} + 2r^2\right] \quad (5)$$

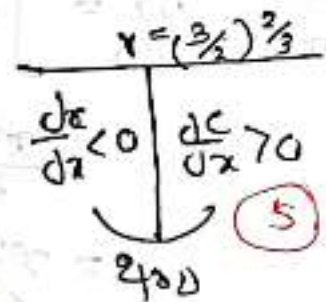
$$\frac{dC}{dr} = 400\pi \left[-\frac{9}{r^2} + 4r\right] \quad (5)$$

$$\frac{dC}{dr} = \frac{400\pi}{r^2} [4r^3 - 9]$$

$$\frac{dC}{dr} = 0 \text{ වන විට } r = \left[\frac{9}{4}\right]^{1/3} \text{ වේ.}$$

$$C \text{ අවම} = 400\pi \left[\frac{9}{\left(\frac{9}{4}\right)^{1/3}} + 2 \cdot \left(\frac{9}{4}\right)^{2/3}\right] \quad (5)$$

$$= \frac{400\pi \left[9 + 2 \cdot \left(\frac{9}{4}\right)^{2/3}\right]}{\left(\frac{13}{4}\right)^{2/3}} \quad (5)$$



70

15) a) $\int \cos 2x \cos 4x \cos 6x dx$

$$I = \frac{1}{2} \int 2 \cos 2x \cos 4x \cos 6x dx$$

$$= \frac{1}{2} \int (\cos 6x + \cos 2x) \cos 6x dx \quad (5)$$

$$= \frac{1}{2} \int \cos^2 6x dx + \frac{1}{2} \int \cos 2x \cos 6x dx$$

$$= \frac{1}{2} \left[\int \frac{1 + \cos 12x}{2} dx \right] + \frac{1}{4} \int (\cos 8x + \cos 4x) dx \quad (5)$$

$$= \frac{1}{2} \left[\frac{1}{2} x + \frac{1}{2} \frac{\sin 12x}{12} \right] + \frac{1}{4} \frac{\sin 8x}{8} + \frac{1}{4} \frac{\sin 4x}{4}$$

$$I = \frac{1}{4} \left[x + \frac{\sin 12x}{12} \right] + \frac{1}{4} \left[\frac{\sin 8x}{8} + \frac{\sin 4x}{4} \right] + C \quad (5) \quad \text{සහ අනිකුත් නියමය}$$

(5)

(5)

(5)

25

$$b) \frac{x^4}{(x-1)(x^2+1)} = Ax+B + \frac{C}{x-1} + \frac{Dx+E}{x^2+1} \quad (5)$$

$$x^4 = (Ax+B)(x-1)(x^2+1) + C(x^2+1) + Dx+E(x-1)$$

$$\left. \begin{array}{l} x^4 \rightarrow 1 = A \\ x^3 \rightarrow 0 = -A+B \\ x^2 \rightarrow 0 = A-B+C+D \\ x \rightarrow 0 = B-A-D+E \\ \text{konst} \rightarrow 0 = -B+C-E \\ \rightarrow 0 = -D+E \end{array} \right\} \begin{array}{l} A=1 \\ B=1 \\ C=\frac{1}{2} \quad (10) \\ E=-\frac{1}{2} \\ D=-\frac{1}{2} \end{array}$$

$$\int \frac{x^4}{(x-1)(x^2+1)} dx = \int (x+1) dx + \int \frac{dx}{2(x-1)} + \int \frac{-\frac{1}{2}x - \frac{1}{2}}{x^2+1} dx \quad (5)$$

$$\begin{aligned} &= \frac{x^2}{2} + x + \frac{1}{2} \ln|x-1| - \frac{1}{4} \int \frac{2x}{x^2+1} - \frac{1}{2} \int \frac{dx}{x^2+1} \\ &= \frac{x^2}{2} + x + \frac{1}{2} \ln|x-1| - \frac{1}{4} \ln|x^2+1| - \frac{1}{2} \tan^{-1} x + C \end{aligned}$$

$$c) \int_{\frac{1}{6}}^{\frac{1}{3}} \frac{\sin 3x}{\sqrt{1-9x^2}} dx \quad \left. \begin{array}{l} \sin 3x = t \quad \text{erprobieren} \quad (5) \\ \frac{3}{\sqrt{1-9x^2}} = \frac{dt}{dx} \\ x = \frac{1}{6}, t = \frac{\pi}{6} \\ x = \frac{1}{3}, t = \frac{\pi}{2} \end{array} \right\} (5)$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{t dt}{3} = \left[\frac{1}{6} [t^2] \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} \quad (5)$$

$$= \frac{1}{6} \left[\left(\frac{\pi}{2}\right)^2 - \left(\frac{\pi}{6}\right)^2 \right] \quad (5)$$

$$= \frac{\pi^2}{6} \left[\frac{1}{4} - \frac{1}{36} \right]$$

$$= \frac{\pi^2}{6} \times \frac{8}{36}$$

$$= \frac{\pi^2}{27} \quad (5)$$

$$d) \int_0^{\pi/2} f(x) dx = \int_0^{\pi/2} f(\pi/2 - x) dx$$

$$\left. \begin{aligned} \pi/2 - x = t, \quad x=0 \text{ so } t = \pi/2 \\ -dx = -dt \quad x = \pi/2 \text{ so } t = 0 \end{aligned} \right\} (5)$$

$$\int_0^{\pi/2} f(\pi/2 - x) dx = \int_{\pi/2}^0 f(t) (-dt)$$

$$= \int_0^{\pi/2} f(t) dt$$

$$= \int_0^{\pi/2} f(x) dx \quad (5)$$

$$= \int_0^{\pi/2} f(x) dx \quad (5)$$

(20)

$$d) I = \int_0^{\pi/2} \frac{x(\sin x + \cos x) dx}{(\cos x - \sin x)^4}$$

$$= \int_0^{\pi/2} \frac{(\pi/2 - x)[\sin(\pi/2 - x) + \cos(\pi/2 - x)] dx}{[\cos(\pi/2 - x) - \sin(\pi/2 - x)]^4}$$

$$= \int_0^{\pi/2} \frac{(\pi/2 - x)(\cos x + \sin x) dx}{(\sin x - \cos x)^4} \quad (5)$$

$$= \int_0^{\pi/2} \frac{\pi/2 [\cos x + \sin x] dx}{(\sin x - \cos x)^4} - \int_0^{\pi/2} \frac{x(\cos x + \sin x) dx}{(\sin x - \cos x)^4}$$

$$= \int_0^{\pi/2} \frac{[\cos x + \sin x] dx}{(\sin x - \cos x)^4} - I$$

$$I = \frac{\pi}{4} \int_0^{\pi/2} \frac{\cos x + \sin x}{(\sin x - \cos x)^4} dx \quad (5)$$

$$= \frac{\pi}{4} \int_0^{\pi/2} (\cos x + \sin x) (\sin x - \cos x)^{-4} dx$$

$$= \frac{\pi}{4} \left[\frac{\sin x - \cos x}{-3} \right]_0^{\pi/2} \quad (5)$$

$$= -\frac{\pi}{12} [3 \sin \pi/2 - \cos \pi/2] - 3 [\sin 0 - \cos 0]^{-3}$$

$$= -\frac{\pi}{12} [1 - (-1)] = -\frac{\pi}{6} \quad (5)$$

(30)

$$b) \frac{x+2y+1}{\sqrt{5}} = \pm \frac{2x+y-4}{\sqrt{5}} \quad (5) \quad (+) \quad x+2y+1 = 2x+y-4$$

$$y-x+5=0 \quad (5)$$

$$x+2y+1 = -(2x+y-4) \quad (5)$$

$$y+x-1=0 \quad (5)$$

$$[2(3)+5-4] [2(4-2-4)]$$

$$7 \times 2 \quad (5)$$

$$14 > 0$$

$\therefore (3,5), (4,-2)$ െരു $2x+y-4=0$ െക്കൊരു ലഘുവക്രമം ഉള്ള
 വൃത്തം കണ്ടു. (5)

$y-x+5=0$ ഉപ $2x+y-4=0$ െക്കൊരു ലഘുവക്രമം
 $y-x+5=0 \rightarrow m_1=1$ െക്കൊരു 2-ആം ലഘുവക്രമം കണ്ടു
 $2x+y-4=0 \rightarrow m_2=-2$ $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| \frac{1 - (-2)}{1 - 2} \right|$

$\tan \theta = 3 \quad (5)$
 $\therefore \tan \theta > 1 \quad (5)$
 $\tan \theta > \tan \frac{\pi}{4}, \theta > \frac{\pi}{4} \quad (5)$

$\therefore y-x+5=0$ െക്കൊരു ലഘുവക്രമം കണ്ടു. (5)
 െന്നു ലഘുവക്രമം കണ്ടു $y+x-1=0 \quad (5)$

50A

ലഘുവക്രമം $S_1 = x^2 + y^2 + 2gx + 2fy + c = 0$

$r = \sqrt{g^2 + f^2 - c}$
 $\sqrt{5} = \sqrt{g^2 + f^2 - c}$
 $g^2 + f^2 - c = 5 \quad (1) \quad (5)$

കേന്ദ്രം $(-g, -f)$, $y+x-1=0$ െക്കൊരു ലഘുവക്രമം
 $-f - g - 1 = 0$
 $g + f + 1 = 0 \quad (2) \quad (5)$

കേന്ദ്രം െക്കൊരു ലഘുവക്രമം കണ്ടു $\sqrt{5}$ െക്കൊരു ലഘുവക്രമം

$\frac{2(-g) - f - 4}{\sqrt{2^2 + 1^2}} = \sqrt{5} \quad (5) \quad -2f - g + 1 = 5$
 $2f + g = -4 \quad (4) \quad (5)$

$\frac{2(-g) - f - 4}{\sqrt{5}} = \sqrt{5}, \quad 2g + f = -9 \quad (3) \quad (5)$

ത്രിഗുണം (3) െക്കൊരു ലഘുവക്രമം $g = -8, f = 7 \quad (5)$
 ത്രിഗുണം (4) െക്കൊരു ലഘുവക്രമം $f = -3, g = 2 \quad (5)$

9 වන & 10 වන අගයන් දුන් ඇති බැවින් අවමයෙන් ආවේණික වන්නේ දන්නා ප්‍රකාරයේ

$g = -8, f = 2x0$

$g = 2, f = -3x0$

① හි $C = (-8)^2 + 7^2 - 5$

① හි $C = 6 - 3^2 + 2^2 - 5$

$C = 108$ (5)

$C = 8$ (5)

∴ අවමයෙන් වන්නේ

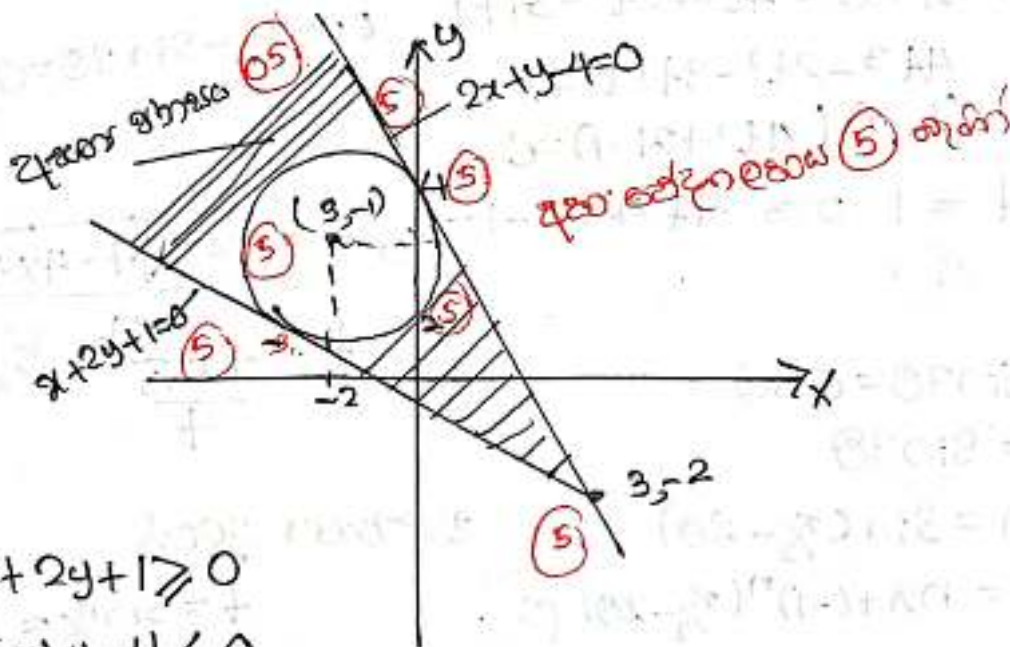
$S_1 = x^2 + y^2 + 2(-8)x + 2(7)y + 108 = 0$

$S_1 = x^2 + y^2 - 16x + 14y + 108 = 0$ (5)

$S_2 = x^2 + y^2 + 2(2x) + 2(-3)y + 8 = 0$

$S_2 = x^2 + y^2 + 4x - 6y + 8 = 0$ (5)

60



$x + 2y + 1 \geq 0$

$2x + y - 4 \leq 0$

$x^2 + y^2 + 4x - 6y + 8 \geq 0$

40

150

$$\begin{aligned}
 17) \cos 2\theta &= \cos(\theta + \theta) = \cos\theta \cos\theta - \sin\theta \sin\theta \\
 &= \cos^2\theta - \sin^2\theta \\
 &= 1 - \sin^2\theta - \sin^2\theta \\
 &= 1 - 2\sin^2\theta \quad (5)
 \end{aligned}$$

$$\begin{aligned}
 \sin 3\theta &= \sin(2\theta + \theta) = \sin 2\theta \cos\theta + \cos 2\theta \sin\theta \\
 &= 2\sin\theta \cos^2\theta + (1 - 2\sin^2\theta) \sin\theta \\
 &= 2\sin\theta (1 - \sin^2\theta) + \sin\theta (1 - 2\sin^2\theta) \\
 &= 3\sin\theta - 4\sin^3\theta \quad (5)
 \end{aligned}$$

$$\begin{aligned}
 \cos 2\theta - \sin 3\theta &= (1 - 2\sin^2\theta) - (3\sin\theta - 4\sin^3\theta) \\
 &= 4\sin^3\theta - 2\sin^2\theta - 3\sin\theta + 1 \quad (5)
 \end{aligned}$$

$t = \sin\theta$ खोलो

$$\cos 2\theta - \sin 3\theta = 4t^3 - 2t^2 - 3t + 1, \quad \cos 2\theta - \sin 3\theta = 0$$

$$4t^3 - 2t^2 - 3t + 1 = 0 \quad (5)$$

$$(t-1)(4t^2 + 2t - 1) = 0$$

$$t = 1 \text{ or } 4t^2 + 2t - 1 = 0, \quad t = \frac{-2 \pm \sqrt{4 - 4 \times 4(-1)}}{2 \times 4} \quad (5)$$

$$t = \frac{-1 \pm \sqrt{5}}{4} \quad (5)$$

$\cos 2\theta - \sin 3\theta = 0$ खोलो

$$\cos 2\theta = \sin 3\theta$$

$$\sin 3\theta = \sin(\frac{\pi}{2} - 2\theta)$$

$$3\theta = n\pi + (-1)^n(\frac{\pi}{2} - 2\theta) \quad (5)$$

अन्यथा ग्राह

$$t = \sin \frac{\pi}{2} = 1$$

$$t = \sin \frac{9\pi}{10} \quad (5)$$

$$t = \sin \frac{13\pi}{10}$$

$n=1$	$n=4$	$n=6$
$\theta = \frac{\pi}{2}$	$\theta = \frac{9\pi}{10}$	$\theta = \frac{13\pi}{10}$

$$\frac{\pi}{2} \leq \theta \leq 3\frac{\pi}{2}$$

$$\theta = \frac{\pi}{2}, \frac{9\pi}{10}, \frac{13\pi}{10} \quad (5)$$

अन्तर्गत $t = 1$ or $t = \frac{-1 \pm \sqrt{5}}{4}$ or

$$\sin \frac{9\pi}{10} > 0$$

$$\sin \frac{13\pi}{10} < 0 \quad (5)$$

$$\therefore \sin \frac{9\pi}{10} = \frac{-1 + \sqrt{5}}{4}$$

$$\frac{x+1+x-1}{1-(x+1)(x-1)} = \frac{8}{31} \quad (5)$$

$$\frac{2x}{1-x^2+1} = \frac{8}{31}$$

$$31x = -4x^2 + 8$$

$$4x^2 + 31x - 8 = 0 \quad (5)$$

$$(x+8)(4x-1) = 0$$

$$x = -8 \text{ or } x = \frac{1}{4} \quad (5)$$

$$\alpha = -8 \neq 0$$

$$\alpha, \beta < 0$$

$$\alpha + \beta < 0$$

$$\therefore \alpha + \beta > 0$$

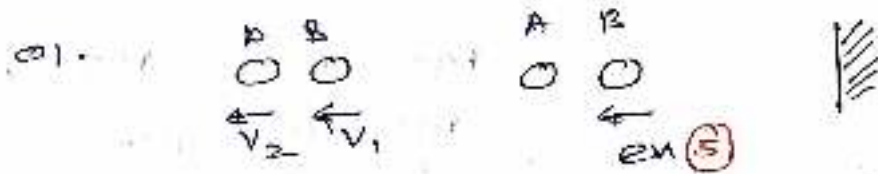
$$\therefore x \neq -8 \quad (5)$$

$$x = \frac{1}{4} \quad (5)$$

1/4

25th Jan 2024
 2024

2024 - II

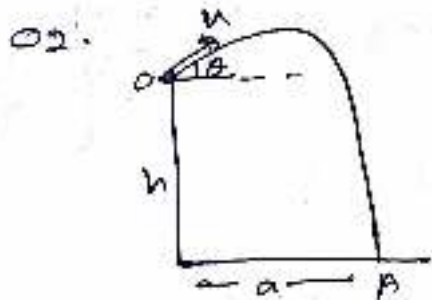


$I = \Delta m v$ (S)
 $m v_1 + m v_2 - m(eu) = 0 \Rightarrow v_1 + v_2 = eu$ (1)

Relative velocity (S)
 $v_2 - v_1 = e^2 u$ (2)

(1) + (2) $v_2 = \frac{e(1+e)u}{2}$ (S) 25

Time = $\frac{a}{2u} + \frac{a}{eu} = \frac{a}{u} \left[\frac{1}{2} + \frac{1}{e} \right]$ (S)



Horizontal distance $s = ut$
 $a = u \cos \theta t \Rightarrow t = \frac{a}{u \cos \theta}$ (S)

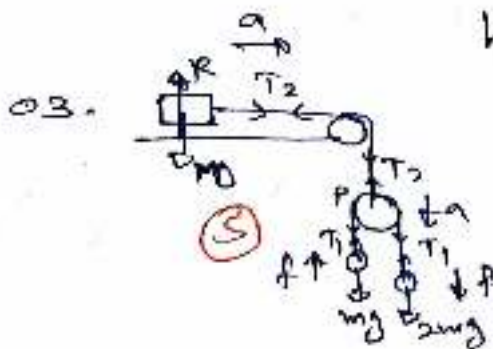
Vertical displacement $s = ut + \frac{1}{2}at^2$
 $-h = u \sin \theta t - \frac{1}{2}gt^2$ (S)

$-h = u \sin \theta \cdot \frac{a}{u \cos \theta} - \frac{1}{2}g \left(\frac{a}{u \cos \theta} \right)^2$

$-h = a \tan \theta - \frac{ga^2}{2u^2} (1 + \tan^2 \theta)$ (S)

$-h = a \cdot \frac{3}{4} - \frac{a}{2} \left(1 + \frac{9}{16} \right) \Rightarrow \frac{3a}{4} - \frac{25a}{32} = \frac{h}{32}$

$h = \frac{a}{32}$ (S) 25



(M) $F = ma \rightarrow T_2 = ma$ (1) (S)

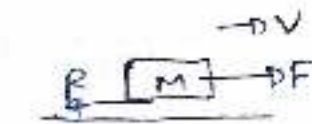
(P) $F = ma \rightarrow 2T_1 - T_2 = 0$ (2) (S)

(m) $F = ma \rightarrow T_1 - mg = m(f-a)$ (3) (S)

(2m) $F = ma \rightarrow 2mg - T_1 = 2m(f+a)$ (4) (S)

25

04.



$$F = ma$$

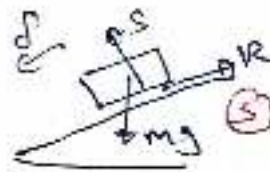
$$F - R = m(0) \quad (5)$$

$$R = F$$

$$H = FV$$

$$F = \frac{H}{V}$$

$$R = \frac{H}{V} \quad (5)$$



$$\sin \alpha = \frac{1}{L}$$

$$F = ma$$

$$mg \sin \alpha - R = mf \quad (5)$$

$$\frac{mg}{L} - \frac{H}{V} = mf$$

$$f = \frac{g}{L} - \frac{H}{mV} \quad (5)$$

25

05.



speed increased gradually $v = 0$

$$\rightarrow (F) F = ma$$

$$T \cos \theta = mR \cos \theta \omega^2 \quad (5)$$

$$\omega = \sqrt{\frac{T}{mR}} \quad (1) \quad \sin \theta = \frac{h}{R}$$

$$(2) \omega = \sqrt{\frac{mgR}{h mR}}$$

$$\uparrow T \sin \theta = mg \quad (5)$$

$$T \cdot \frac{h}{R} = mg \Rightarrow T = \frac{mgR}{h} \quad (5)$$

$$\omega = \sqrt{\frac{g}{h}} \quad (5)$$

25

$$06. \vec{OA} \perp \vec{OB} \Rightarrow \vec{OA} \cdot \vec{OB} = 0 \quad (5)$$

$$(\underline{a} - 2\underline{b}) \cdot (3\underline{a} + \underline{b}) = 0$$

$$3\underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{b} - 6\underline{a} \cdot \underline{b} - 2\underline{b} \cdot \underline{b} = 0$$

$$3|\underline{a}|^2 - 2|\underline{b}|^2 = 5\underline{a} \cdot \underline{b} \quad (5)$$

$$\underline{a} \cdot \underline{b} = \frac{3}{5}|\underline{a}|^2 - \frac{2}{5}|\underline{b}|^2$$

$$\underline{a} \cdot \underline{b} = \frac{3}{5}(1)^2 - \frac{2}{5}(2)^2 = -1 \quad (5)$$

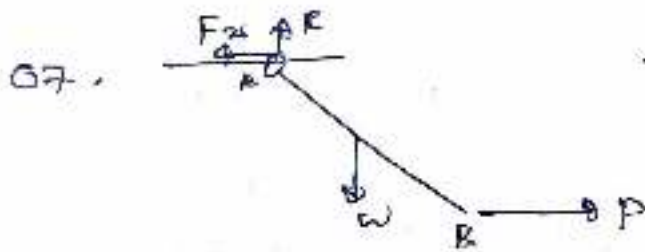
$$\underline{a} \cdot \underline{b} = |\underline{a}| |\underline{b}| \cos \theta \quad (5)$$

$$1 \cdot 2 \cos \theta = -1$$

$$\cos \theta = -\frac{1}{2} \Rightarrow \theta = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = \frac{3\pi}{2} \quad (5)$$

25



$$\rightarrow P = F_{2c} \quad \uparrow R = W \quad \text{(5)}$$

$$\sum \tau_A = 0 \quad 2a \cos \theta \cdot P - a \sin \theta \cdot W = 0 \quad \text{(5)}$$

$$\frac{2P}{W} = \frac{\sin \theta}{\cos \theta}$$

$$\tan \theta = \frac{2P}{W}$$

$$\tan \theta = 2 \frac{F_{2c}}{R} \quad \text{(5)}$$

ଅବଶ୍ୟକୀୟ ଶକ୍ତି,

$$\mu \geq \frac{|F_{2c}|}{R} \quad \text{(5)}$$

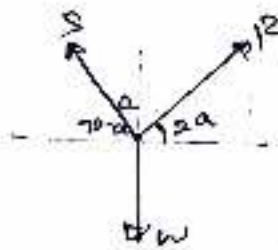
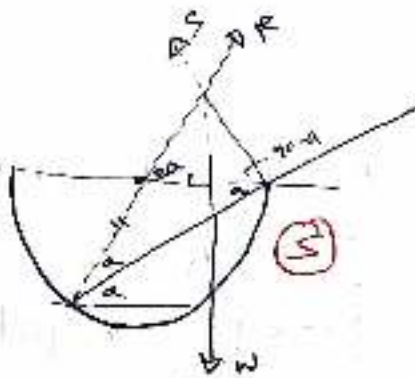
$$\mu \geq \frac{\tan \theta}{2}$$

$$2\mu \geq \tan \theta$$

$$\theta \leq \tan^{-1}(2\mu) \quad \text{(5)}$$

25

08.



$$\frac{S}{\sin(90+2a)} = \frac{R}{\sin(180-a)} = \frac{W}{\sin(90-a)}$$

$$\frac{S}{\cos 2a} = \frac{R}{\sin a} = \frac{W}{\cos a} \quad \text{(10)}$$

$$S = W \frac{\cos 2a}{\cos a} \quad \text{(5)}$$

$$R = W \frac{\sin a}{\cos a} = W \tan a \quad \text{(5)}$$

25

$$09. P(A) = \frac{1}{3} \quad P(B) = \frac{3}{5}$$

$$P(A \cap B) = \frac{2}{5}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} \quad \text{(5)} \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = \frac{2}{5} \times \frac{1}{3} = \frac{1}{5} \quad \text{(5)}$$

$$= \frac{1}{3} + \frac{3}{5} - \frac{1}{5}$$

$$= \frac{11}{15} \quad \text{(5)}$$

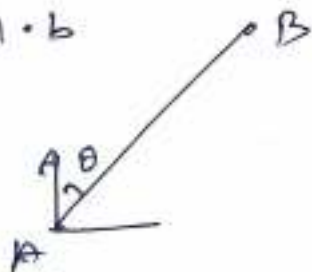
$$P(B) = P(A \cap B) + P(A' \cap B) \quad \text{(5)}$$

$$= \frac{1}{5} + \frac{2}{5}$$

$$= \frac{3}{5} \quad \text{(5)}$$

25

11.6



$$V_{KS} = \frac{V}{K} \quad (5)$$

$$V_{SE} = \frac{V}{E} \quad (5)$$

$$V_{KE} = \frac{V}{S} \quad (5)$$

K - 2000

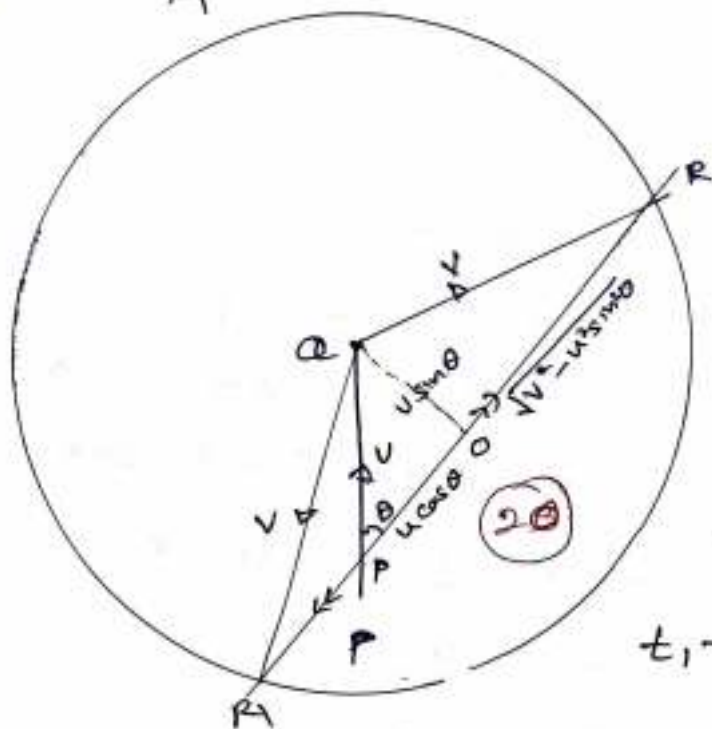
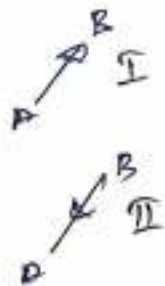
E - 2000

S - 2000

$$V_{KS} = V_{KE} + V_{ES}$$

$$\frac{V}{K} = \frac{V}{E} + \frac{V}{S} \quad (10)$$

$$= \frac{V}{K} + \frac{V}{S}$$



$$OQ = U \sin \theta$$

$$PO = U \cos \theta$$

$$OR = \sqrt{V^2 - U^2 \sin^2 \theta} \quad (5)$$

$$PR = \sqrt{V^2 - U^2 \sin^2 \theta} + U \cos \theta \quad (5)$$

$$PR_1 = \sqrt{V^2 - U^2 \sin^2 \theta} - U \cos \theta \quad (5)$$

$$t_1 = \frac{a}{PR} \quad t_2 = \frac{a}{PR_1}$$

$$t_1 - t_2 = \frac{a}{PR} - \frac{a}{PR_1}$$

$$= \frac{a PR_1 - a PR}{PR \cdot PR_1} \quad (10)$$

$$t_1 - t_2 = \frac{a \sqrt{V^2 - U^2 \sin^2 \theta} - U a \cos \theta - a \sqrt{V^2 - U^2 \sin^2 \theta} - U a \cos \theta}{(\sqrt{V^2 - U^2 \sin^2 \theta} + U \cos \theta)(\sqrt{V^2 - U^2 \sin^2 \theta} - U \cos \theta)}$$

$$= \frac{-2U a \cos \theta}{V^2 - U^2 \sin^2 \theta - U^2 \cos^2 \theta} = \frac{-2U a \cos \theta}{V^2 - U^2 (\sin^2 \theta + \cos^2 \theta)}$$

$$= \frac{-2U a \cos \theta}{V^2 - U^2}$$

$$= \frac{-2U a \cos \theta}{K^2 U^2 - U^2}$$

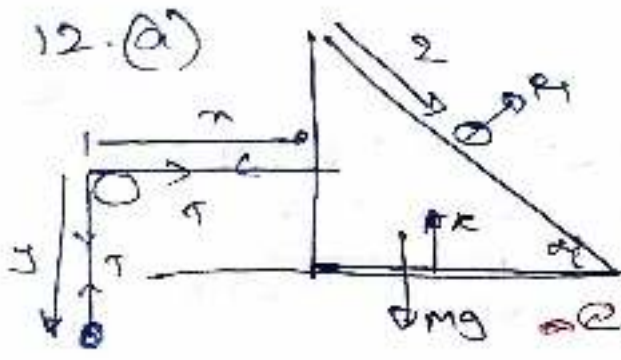
$$= \frac{-2a \cos \theta}{U(K^2 - 1)} \quad (10)$$

V m u zy-e-ta zam

V a b (5)

V = KU odu info

80



$$a_{ME} = \ddot{x}$$

$$a_{mM} = \ddot{y} \quad (10)$$

$$a_{m,E} = a_{m,M} + a_{ME}$$

$$= \ddot{y} + \ddot{x}$$

$$a_{\lambda m,E} = \ddot{y}$$

$\lambda m \downarrow F = ma$

$$\lambda mg - T = \lambda m \ddot{y} \quad (1)$$

$$x + y = l$$

$$\dot{x} + \dot{y} = 0$$

$$\ddot{x} + \ddot{y} = 0$$

$$\ddot{x} = -\ddot{y} \quad (4)$$

$m, M \rightarrow F = ma$

$$-T = M \ddot{x} + m(\ddot{x} + \ddot{y} \cos \alpha) \quad (2)$$

$m \downarrow F = ma$

$$mg \sin \alpha = m(\ddot{y} + \ddot{x} \cos \alpha) \quad (3)$$

$$(2) - (1) \quad -\lambda mg = m \ddot{x} + m(\ddot{x} + \ddot{y} \cos \alpha) - \lambda m \ddot{y}$$

$$= (M + m + \lambda m) \ddot{x} + m(\ddot{y} \cos \alpha - \ddot{x})$$

$$\ddot{x} = \frac{-mg(1 + \sin \alpha \cos \alpha)}{M + \lambda m + m \sin^2 \alpha} \quad (5)$$

$$T = \lambda m(g + \ddot{y})$$

$$= \lambda m \left[g - \frac{mg(1 + \sin \alpha \cos \alpha)}{M + \lambda m + m \sin^2 \alpha} \right] \quad (5)$$

$$= \lambda mg \left[\frac{M + \lambda m + m \sin^2 \alpha - mg(1 + \sin \alpha \cos \alpha)}{M + \lambda m + m \sin^2 \alpha} \right] \quad (5)$$

$$= \lambda mg \left[\frac{M + m \sin \alpha (\sin \alpha - \cos \alpha)}{M + m(1 + \sin^2 \alpha)} \right] \quad (5)$$

12-b



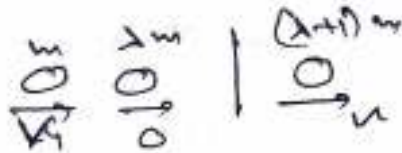
$$0 - mga \sin \alpha = \frac{1}{2} m v_1^2 - mga \quad (5)$$

$$v_1^2 = 2ga(1 - \sin \alpha)$$

$$v_1 = \sqrt{2ga(1 - \sin \alpha)} \quad (5)$$

10

(i)

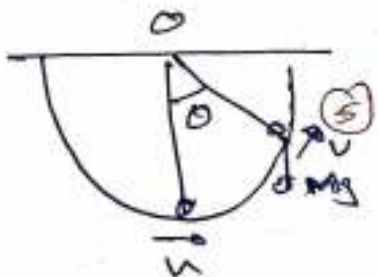


$$I = \lambda m v \rightarrow m v_1 = (\lambda + 1) m u \quad (5)$$

$$u = \frac{\sqrt{2ga(1 - \sin \alpha)}}{\lambda + 1} \quad (5)$$

10

(ii)



$$M = (\lambda + 1) m$$

$$\frac{1}{2} m u^2 - mga = \frac{1}{2} m v^2 - mga \cos \theta \quad (10)$$

$$v^2 = u^2 - 2ga + 2ga \cos \theta$$

$$= \frac{2ga(1 - \sin \alpha)}{(\lambda + 1)^2} - 2ga(1 - \cos \theta)$$

$$v^2 = \frac{2ga}{(\lambda + 1)^2} [1 - \sin \alpha - (\lambda + 1)^2 (1 - \cos \theta)]$$

$$v = \frac{1}{(\lambda + 1)} \sqrt{2ga [1 - \sin \alpha - (\lambda + 1)^2 (1 - \cos \theta)]} \quad (5)$$

20

(iii) $v = 0$ so $\theta = \theta_1$

$$\frac{1}{\lambda + 1} \sqrt{2ga [1 - \sin \alpha - (\lambda + 1)^2 (1 - \cos \theta_1)]} = 0 \quad (5)$$

$$1 - \sin \alpha - (\lambda + 1)^2 (1 - \cos \theta_1) = 0$$

$$\cos \theta_1 = 1 - \frac{1 - \sin \alpha}{(\lambda + 1)^2} \Rightarrow \theta = \cos^{-1} \left[1 - \frac{1 - \sin \alpha}{(\lambda + 1)^2} \right] \quad (5)$$

10

(iv)

$$F = ma$$

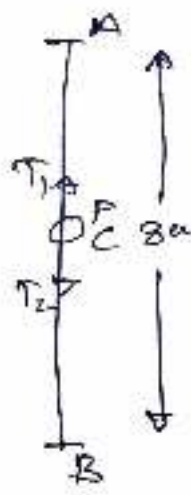
$$R - mg \cos \theta = \frac{m v^2}{a}$$

$$R = mg \cos \theta = Mg \left[1 - \frac{1 - \sin \alpha}{(\lambda + 1)^2} \right] \quad (10)$$

$$= (\lambda + 1) mg \left[1 - \frac{1 - \sin \alpha}{(\lambda + 1)^2} \right] \quad (5)$$

20

13



$$T_1 = \frac{mg(AC-a)}{a} \quad T_2 = \frac{mg(2a-AC-2a)}{3a}$$

$$= \frac{mg(5a-AC)}{3a}$$

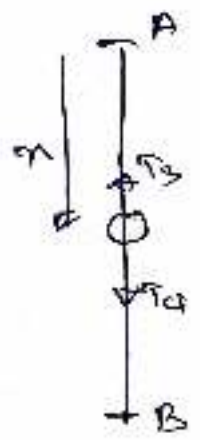
$\uparrow F = ma$

$$T_1 = T_2 + mg \quad (10)$$

$$\frac{mg(AC-a)}{a} = \frac{mg(5a-AC)}{3a} + mg \quad (5)$$

$$3(AC-a) = 5a - AC + 3a$$

$$AC = \frac{11a}{4} \quad (5) \quad \boxed{30}$$



$$T_3 = \frac{mg(n-a)}{a} \quad T_4 = \frac{mg(5a-n)}{3a}$$

$\uparrow F = ma$

$$T_3 - T_4 - mg = m\ddot{n} \quad (10)$$

$$\frac{mg(n-a)}{a} - \frac{mg(5a-n)}{3a} - mg = -m\ddot{n} \quad (5)$$

$$\frac{g}{3a} [3n - 3a - 5a + n - 3a] = -\ddot{n}$$

$$\ddot{n} + \frac{4g}{3a} [n - \frac{11a}{4}] = 0 \quad (5)$$

$$\ddot{y} + \frac{4g}{3a} y = 0 \quad (5) \quad \text{--- (A)}$$

$$y = n - \frac{11a}{4}$$

$$\ddot{y} = \ddot{n}$$

$$\ddot{y} = -\ddot{n} \quad (5)$$

of (10)

$$y = A \cos \omega t + B \sin \omega t$$

$$\dot{y} = -A\omega \sin \omega t + B\omega \cos \omega t$$

$$\ddot{y} = -A\omega^2 \cos \omega t - B\omega^2 \sin \omega t$$

$$= -\omega^2 (A \cos \omega t + B \sin \omega t)$$

$$\ddot{y} = -\omega^2 y$$

$$\ddot{y} + \omega^2 y = 0 \quad \text{--- (B) (5)}$$

(A) & (B) gives

$$\omega = \sqrt{\frac{4g}{3a}} \quad (5)$$

$$n = 5a, \quad t = 0 \quad (5)$$

$$y = 5a - \frac{11a}{4} = \frac{9a}{4}$$

$$\frac{9a}{4} = A \cos 0 + B \sin 0$$

$t = 0 \Rightarrow \dot{y} = 0 \Rightarrow \dot{n} = 0$

$$0 = -A\omega \sin 0 + B\omega \cos 0 \Rightarrow B = 0 \quad \boxed{45}$$

$$x = a$$

$$y = a - \frac{11a}{4} = -\frac{7a}{4} \quad (5)$$

$$-\frac{7a}{4} = \frac{9a}{4} \cos \omega t_1$$

$$\cos \omega t_1 = -\frac{7}{9} \quad (5)$$

$$\cos \omega t_1 = \cos(\pi - \theta) \quad ; \cos \theta = \frac{7}{9} \quad (5)$$

$$\omega t_1 = \pi - \theta \quad (5)$$

$$\sin \theta = \frac{4\sqrt{2}}{9}$$

$$y = -\frac{9a}{4} \sqrt{\frac{49}{3a}} \sin(\pi - \theta)$$

$$= -\frac{9a}{4} \sqrt{\frac{49}{3a}} \times \frac{4\sqrt{2}}{9}$$

$$= \sqrt{\frac{89a}{3}} \quad (5)$$

$$\therefore \text{part } \textcircled{1} \text{ answer} = \sqrt{\frac{89a}{3}} \quad (5)$$

$$\omega t_1 = \pi - \theta$$

$$t_1 = \frac{\pi - \theta}{\omega}$$

$$t = \sqrt{\frac{3a}{49}} \left[\pi - \cos^{-1}\left(\frac{7}{9}\right) \right] \quad (5)$$

35



S.H.M is

$$x = \frac{11a}{4} \quad (5)$$

Amplitude is

$$5a - \frac{11a}{4} = \frac{9a}{4} \quad (5)$$

$$\cos \theta = \frac{7a/4}{9a/4}$$

$$\theta = \cos^{-1}\left(\frac{7}{9}\right) \quad (5)$$

$$v^2 = \omega^2(a^2 - x^2)$$

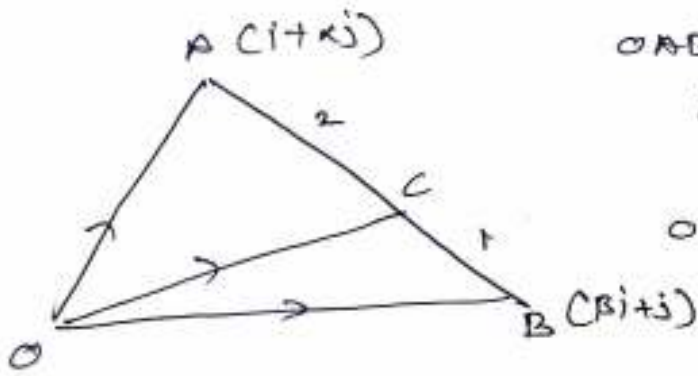
$$= \frac{49}{3a} \left[\left(\frac{11a}{4}\right)^2 - \left(\frac{7a}{4}\right)^2 \right] \quad (5)$$

$$= \sqrt{\frac{89a}{3}} \quad (5)$$

$$T = \frac{\pi - \theta}{\omega} = \sqrt{\frac{3a}{49}} \left[\pi - \cos^{-1}\left(\frac{7}{9}\right) \right] \quad (5)$$

35

14. a Theorem - (10)



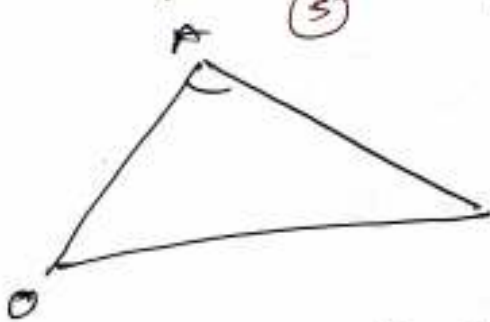
OABD का,
 $\vec{AB} = \vec{AO} + \vec{OB}$
 $= (\beta-1)\underline{i} + (1-k)\underline{j}$ (5)

OACD का,
 $\vec{OC} = \vec{OA} + \vec{AC}$
 $= \vec{OA} + \frac{2}{3}\vec{AB}$ (5)
 $= \underline{i} + k\underline{j} + \frac{2}{3}((\beta-1)\underline{i} + (1-k)\underline{j})$ (5)
 $= \frac{(2\beta+1)\underline{i} + (\alpha+2)\underline{j}}{3}$ (5)

OADC समतलचतुर्भुज का है,
 $\vec{OC} = \vec{AD}$

$\left(\frac{2\beta+1}{3}\right)\underline{i} + \left(\frac{\alpha+2}{3}\right)\underline{j} = 3\underline{i} + \underline{j}$ (5)
 $\frac{2\beta+1}{3} = 3 \quad , \quad \frac{\alpha+2}{3} = 1 \quad ; \quad \underline{i} \neq 0 \quad \underline{j} \neq 0$ (5)
 $\underline{i} \times \underline{j}$

$\beta = 4 \quad , \quad \alpha = 1$ (5)



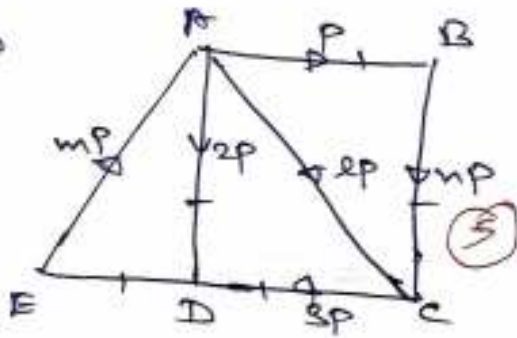
$|\vec{OA}| = |\underline{i} + \underline{j}| = \sqrt{2}$
 $|\vec{AB}| = |\underline{3i}| = 3$ (10)
 $|\vec{OB}| = |\underline{4i} + \underline{j}| = \sqrt{17}$

$\cos A = \frac{(\vec{OA})^2 + (\vec{AB})^2 - (\vec{OB})^2}{2(\vec{OA})(\vec{AB})}$ (5)

$\cos A = -\frac{1}{\sqrt{2}}$ (5)

75

14.b



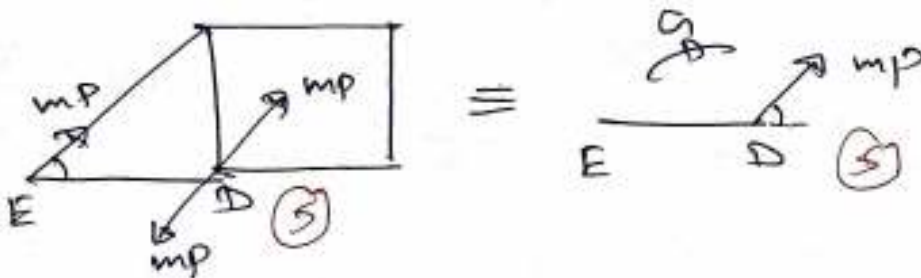
આ સંરુચીય સંતુલન સ્થિતિ

$$\sum \tau_A = 0 \quad nP \times a + 3P \times a = 0 \Rightarrow n = -3 \quad (5)$$

$$\sum \tau_B = 0 \quad P - 3P + \frac{mP}{\sqrt{2}} + \frac{lP}{\sqrt{2}} = 0 \Rightarrow m + l = 2\sqrt{2} \quad (5)$$

$$\sum \tau_C = 0 \quad 3P - 2P + \frac{mP}{\sqrt{2}} - \frac{lP}{\sqrt{2}} = 0 \Rightarrow l - m = \sqrt{2} \quad (5)$$

$$l = \frac{3\sqrt{2}}{2} \quad m = \frac{\sqrt{2}}{2} \quad (10)$$



અનુસર્ય સંતુલન = $\frac{mP}{\sqrt{2}} \cdot a \quad (5)$

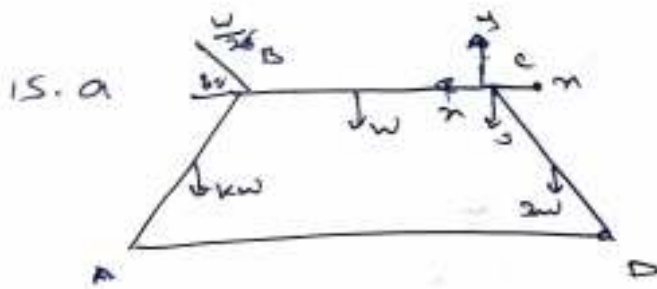
$$\tau = a \cdot \frac{\sqrt{2}}{2} \cdot P$$

$$= \frac{aP}{2} \quad (5)$$

સંતુલન સ્થિતિ સંતુલન સૂત્રો વાપરે છે જે અનુસર્ય સૂત્રો છે. ABCD છે.

Q (5)

75



⑤

$$W \times a = y \times 2a$$

$$y = \frac{W}{2} \quad (5)$$

⑩

$$3W \times \frac{a}{2} + W \times \frac{2a}{2} = x \times 2a \frac{\sqrt{3}}{2a}$$

$$\frac{3W}{2} + \frac{W}{2} = \sqrt{3} x$$

$$x = \frac{2W}{\sqrt{3}}$$

ABC centroid

⑩

$$Kw \frac{a}{2} + \frac{W}{2\sqrt{3}} \times \frac{1}{2} \times \sqrt{3} \times 2a + \frac{a}{2\sqrt{3}} \times \frac{\sqrt{3}}{2} \times \frac{2a}{2} + W \left(\frac{2a}{2} + a \right)$$

$$= \frac{2\sqrt{3}}{\sqrt{3}} W \times 2a \times \frac{\sqrt{3}}{2} + \frac{W}{2} \times 2a \quad (10)$$

$$\frac{Kw}{2} + \frac{W}{4} + \frac{W}{4} + 2W = 2W + \frac{3W}{2}$$

$$\frac{Kw}{2} = \frac{3W}{2} - \frac{W}{2}$$

$$\frac{Kw}{2} = W$$

$$K = 2 \quad (5)$$

⑩

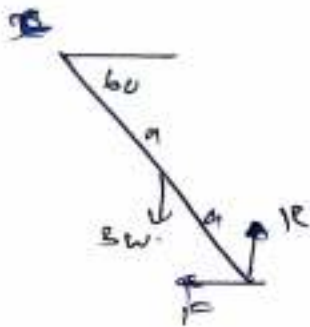
$$Kw \cdot \frac{a}{2} + \frac{W}{2\sqrt{3}} \times \frac{1}{2} \times 2a \times \frac{\sqrt{3}}{2} + \frac{a}{2\sqrt{3}} \times \frac{\sqrt{3}}{2} \times \frac{2a}{2} + W \left(\frac{2a}{2} + a \right) = R + 4a$$

$$+ 3W \left(2a \times \frac{1}{2} + 2a + \frac{a}{2} \right)$$

$$W + \frac{W}{4} + \frac{W}{4} + 2W + 3W \times \frac{7}{2} = 4R$$

$$4R = \frac{4W + W + W + 8W + 42W}{4} = 14W$$

$$R = \frac{14W}{4} = \frac{7W}{2} \quad (5)$$



(c)

$$3W \cdot \frac{a}{2} + F \times 2a\sqrt{3} = R \times 2a \times \frac{1}{2}$$

$$\sqrt{3} F = R - \frac{3W}{2}$$

$$F = \frac{7W}{2} - \frac{3W}{2} = 2W$$

$$F = \frac{2W}{\sqrt{3}} \quad (5)$$

2020-2021

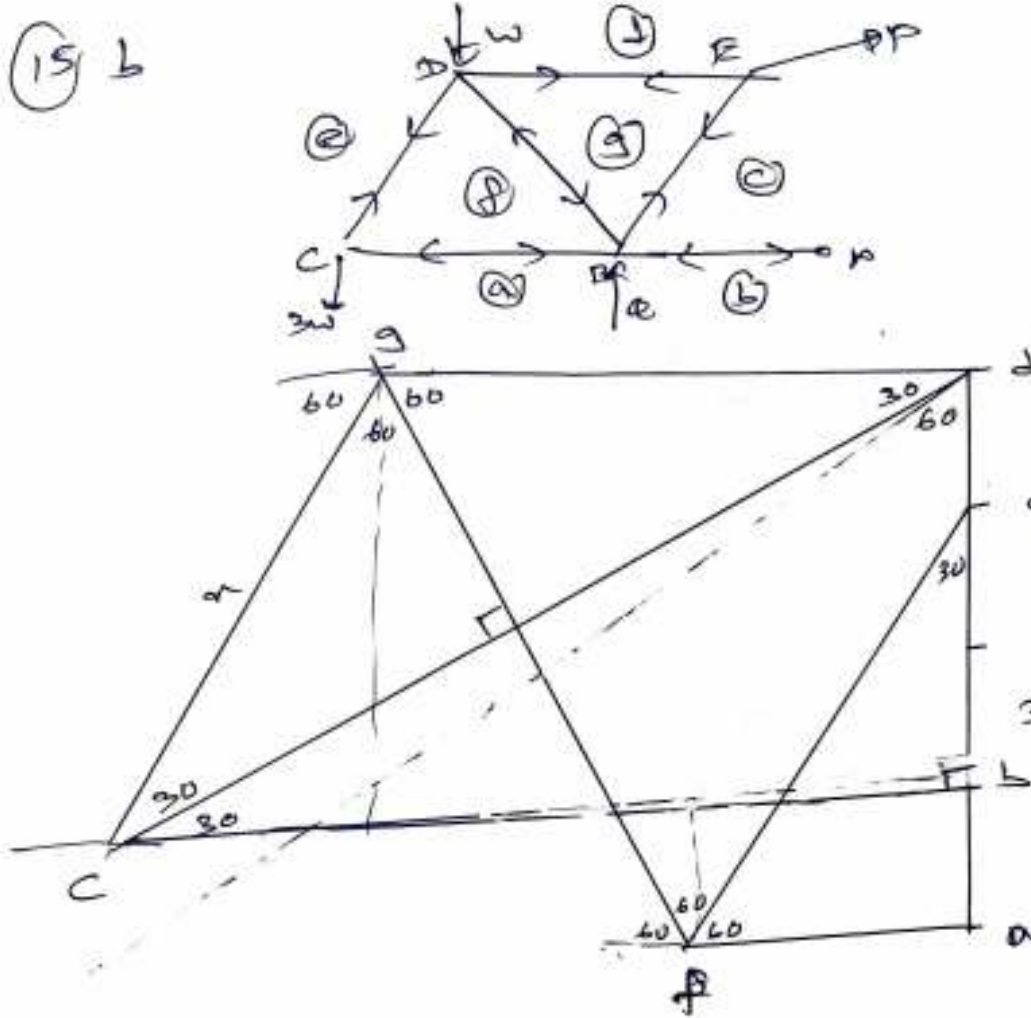
$$\mu \geq \frac{F}{R}$$

$$\mu \geq \frac{2W/\sqrt{3}}{\frac{7W}{2}} \quad (5) \Rightarrow \frac{4W}{7\sqrt{3}W} = \frac{4\sqrt{3}}{21} \quad (5)$$

$$\mu \geq \frac{4\sqrt{3}}{21}$$

65

(15) b



- 2nd 5 sets
- 4 - 30
- 3 - 20
- 2 - 15
- 1 - 10

$P \rightarrow ab \Rightarrow w$ (5)
 $Q = cd \Rightarrow 6w$ (5)

$n \cos 30 = 3w$
 $n \frac{\sqrt{3}}{2} = 3w$
 $n = 2\sqrt{3}w$

$2n \cos 30 = CD$
 $2 \cdot 2\sqrt{3}w \cdot \frac{\sqrt{3}}{2} = CD$
 $CD = 6w$

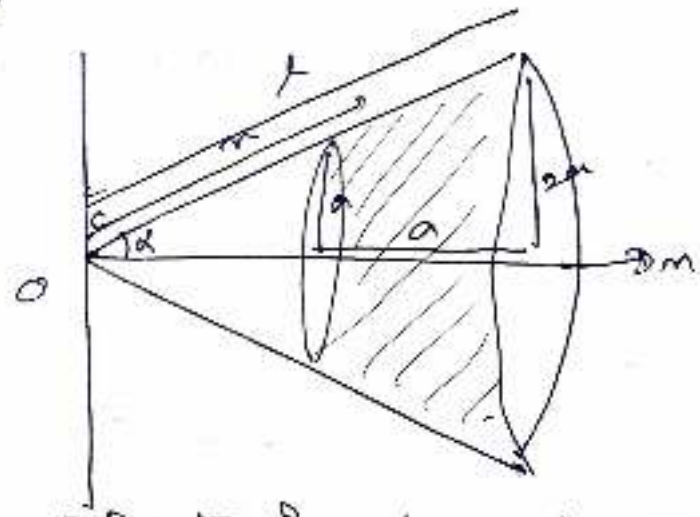
एडी	समान	असमान
AB (bc)	✓	
BC (af)	✓	
CD (ef)	✓	
DE (dg)	✓	
BE (eg)	✓	
BD (fg)	✓	

- $3\sqrt{3}w$ (5)
- $\sqrt{3}w$ (5)
- $2\sqrt{3}w$ (5)
- $2\sqrt{3}w$ (5)
- $2\sqrt{3}w$ (5)
- $\frac{8w}{\sqrt{3}}$ (5)

2nd 5 sets (15)

85

16



$$\tan \alpha = \frac{2a}{2a} = 1$$

$$a = \frac{2a}{4}$$

$$dm = 2(\pi r \tan \alpha) \frac{\rho}{a} dx$$

$$M = \int_a^{2a} \pi r^2 \frac{\rho}{a} dx$$

$$= \frac{\pi \rho}{a} \int_a^{2a} r^2 dx$$

$$= \frac{\pi \rho}{a} \left[\frac{r^3}{3} \right]_a^{2a}$$

$$= \frac{\pi \rho}{a} \left[\frac{8a^3 - a^3}{3} \right]$$

$$= \frac{7\pi a^3 \rho}{3}$$

ଅକ୍ଷରୁ ଦୂରତାର ଉତ୍ତମ ମାନ
 ନିମ୍ନରେ ଦିଆଯାଇଥିବା ସୂତ୍ର (5)

$$\bar{x} = \frac{\int_a^{2a} x \tan^2 \alpha \cdot \pi r^2 \frac{\rho}{a} dx}{\int_a^{2a} \pi r^2 \frac{\rho}{a} dx}$$

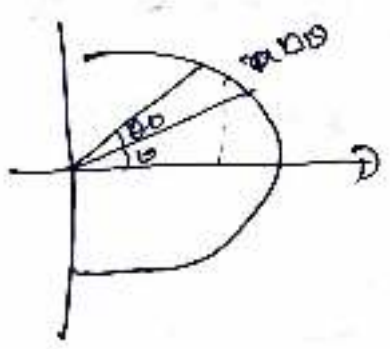
$$= \frac{\int_a^{2a} x^3 dx}{\int_a^{2a} x^2 dx} = \frac{\left[\frac{x^4}{4} \right]_a^{2a}}{\left[\frac{x^3}{3} \right]_a^{2a}}$$

$$= \frac{\frac{16a^4 - a^4}{4}}{\frac{8a^3 - a^3}{3}} = \frac{3}{4} \times \frac{15a^4}{7a^3} = \frac{45}{28} a$$

(0 ରୁ 45/28 36)

କୂଳୀ ମୂଳିକା କିମ୍ବା 36 = $\frac{45}{28} a - a$

$$= \frac{17}{28} a$$

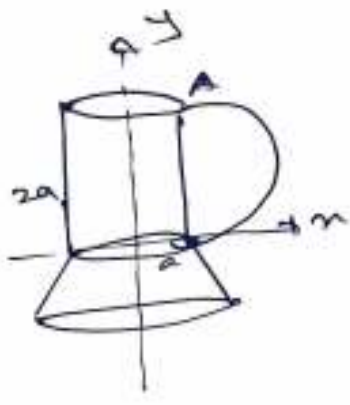


ଅକ୍ଷରୁ ଦୂରତାର ଉତ୍ତମ ମାନ
 ନିମ୍ନରେ ଦିଆଯାଇଥିବା ସୂତ୍ର (10) Δ ମା $a \sin \theta$

$$\bar{x} = \frac{\int_{-\pi/2}^{\pi/2} x \rho a \cos \theta d\theta}{\int_{-\pi/2}^{\pi/2} \rho a d\theta} = \frac{a \left[\sin \theta \right]_{-\pi/2}^{\pi/2}}{\left[\theta \right]_{-\pi/2}^{\pi/2}}$$

$$= \frac{a(1 - (-1))}{\pi - (-\pi)} = \frac{2a}{\pi}$$

~~SS~~ SS =



ଅଂଶ	ଅନୁପାତ	କ୍ଷେତ୍ର	ଉଚ୍ଚତା	ଜ୍ୟାମିତି
	$2a^2\rho$ (3)	a (3)	0 (3)	0 (3)
	$2a^2\rho$ (3)	a (3)	$a + \frac{2a}{2}$ (3)	0 (3)
	$\frac{7a^2\rho}{3}$ (3)	$-\frac{17}{28}a$ (3)	0 (3)	0 (3)
ସମସ୍ତ	$\frac{13a^2\rho}{3} + 2a^2\rho$ (3)	\bar{y}	\bar{x}	
	$2a^2\rho(\frac{13}{3}\rho + 6)$			

$$\bar{x} = \frac{2a^2\rho(a + \frac{2a}{2})}{2a^2\rho(\frac{13}{3}\rho + 6)} = \frac{a\rho(1 + \frac{2}{2})}{\frac{13}{3}\rho + 6} \quad (9)$$

$$\bar{y} = \frac{2a^2\rho + 2a^2\rho - \frac{17a^2\rho}{12}}{2a^2\rho(\frac{13}{3}\rho + 6)} \quad (10)$$

$$= \frac{a(6 + \frac{7}{12}\rho)}{\frac{13}{3}\rho + 6} \quad (5)$$

A ଚର୍ଚ୍ଚିତ କର୍ଣ୍ଣ

$$\tan \alpha = \frac{a - \left[\frac{a\rho(1 + \frac{2}{2})}{\frac{13}{3}\rho + 6} \right]}{2a - \left[\frac{a(6 + \frac{7}{12}\rho)}{\frac{13}{3}\rho + 6} \right]} \quad (5)$$

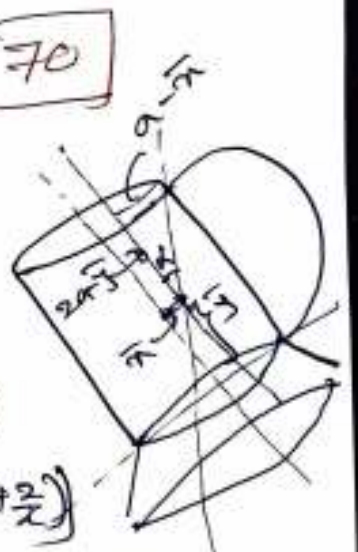
$$\frac{1}{2} \left[2a\left(\frac{13}{3}\rho + 6\right) - a(6 + \frac{7}{12}\rho) \right] = \left[a\left(\frac{13}{3}\rho + 6\right) - a(6 + \frac{2}{2}) \right]$$

$$2a\left(\frac{13}{3}\rho + 6\right) - a(6 + \frac{7}{12}\rho) = 2a\left(\frac{13}{3}\rho + 6\right) - 2a(6 + \frac{2}{2})$$

$$\frac{2\rho}{2} \times 26 + 4a = 26 + \frac{7\rho}{12} \times a \quad (10)$$

$$\frac{26 + 4a}{2} = \frac{7\rho}{12}$$

$$12(26 + 4a) = 7\rho \times 2 \quad (10)$$



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17. a

N - නිමැවී විකිණන දැවැන්ත $P(N) = \frac{1}{2}$

K - කැමැති විකිණන දැවැන්ත $P(K) = \frac{1}{3}$

S - ප්‍රකෘති විකිණන දැවැන්ත $P(S) = \frac{1}{6}$

A - සමහරක් විකිණන දිනි තිබීම

$$P\left(\frac{A}{N}\right) = \frac{5}{100} \quad P\left(\frac{A}{K}\right) = \frac{6}{100} \quad P\left(\frac{A}{S}\right) = \frac{8}{100}$$

සමහරක් විකිණන දිනි තිබීමේ අවස්ථාව $P(A)$ වේ

$$P(A) = P\left(\frac{A}{N}\right)P(N) + P\left(\frac{A}{K}\right)P(K) + P\left(\frac{A}{S}\right)P(S) \quad (10)$$

$$= \frac{5}{100} \times \frac{1}{2} + \frac{6}{100} \times \frac{1}{3} + \frac{8}{100} \times \frac{1}{6}$$

$$= \frac{35}{600} = \frac{7}{120}$$

$$\text{I} \quad P\left(\frac{N}{A}\right) = \frac{P\left(\frac{A}{N}\right) \cdot P(N)}{P(A)} = \frac{\frac{5}{100} \cdot \frac{1}{2}}{\frac{7}{120}} = \frac{3}{7} \quad (10)$$

$$\text{II} \quad P\left(\frac{K}{A}\right) = \frac{P\left(\frac{A}{K}\right) \cdot P(K)}{P(A)} = \frac{\frac{6}{100} \times \frac{1}{3}}{\frac{7}{120}} = \frac{12}{35} \quad (10)$$

$$\text{III} \quad P\left(\frac{S}{A}\right) = \frac{P\left(\frac{A}{S}\right) \cdot P(S)}{P(A)} = \frac{\frac{8}{100} \cdot \frac{1}{6}}{\frac{7}{120}} = \frac{8}{35} \quad (10)$$

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Interval	x_i	f_i	d	f_d	x_i^2	$f_i x_i^2$
0000-2000	1000	10	-4000	-40000	1×10^6	10×10^6
2000-4000	3000	15	-2000	-30000	9×10^6	135×10^6
4000-6000	5000	40	0	0	25×10^6	1000×10^6
6000-8000	7000	20	2000	40000	49×10^6	980×10^6
8000-10000	9000	15	4000	60000	81×10^6	1215×10^6
		<u>100</u>		<u>30000</u>		<u>3340×10^6</u>

Σ $f_i x_i = 30000$ (10)

$$I. \mu = \frac{5000 + \frac{30000}{100}}{100} = 5300 \text{ (Ans)} \quad (10)$$

$$II. M_2 = 4000 + \frac{2000}{40} \left(\frac{100}{2} - 25 \right) = 5250 \text{ (Ans)} \quad (10)$$

$$III. S = \sqrt{\frac{3340 \times 10^6}{100} - (5300)^2} \quad (10)$$

$$= \sqrt{531 \times 10^2}$$

$$= 23.1 \times 10^2$$

$$= \underline{\underline{2310}} \quad (5)$$

$$IV. \mu = 5300 \text{ అనేది అభివృద్ధి అనుబంధం యొక్క సగటు.} \quad (10)$$

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