සංයුක්ත ගණිතය II.

පිළිතුරු පත්‍රය.

01. $\underline{a}, \underline{b} \neq 0$ සහ $\underline{a} \nparallel \underline{b}$

$\lambda a + \mu b = 0$ —— ①

නමුත් $\lambda \neq 0$ නම්, එවිට $\underline{a} = -\frac{\mu}{\lambda}\underline{b}$. 5

මෙය දුන් ළද අවශ්‍යතාවයට පටහැනි වේ. 5

$\therefore \lambda = 0$ ① න්, $\mu b = 0$ ලැබේ. 5

$\underline{b} \neq 0$ නිසා $\mu = 0$ 5

$\therefore \lambda = 0$ හා $\mu = 0$ වේ. 5

02. $\underline{a} \perp \underline{b}$ නම් $\underline{a} \cdot \underline{b} = 0 \Rightarrow (2\underline{i} + p\underline{j}) \cdot (2\underline{i} - 5\underline{j}) = 0$ 5

$4 - 5p = 0$

$p = 4/5$ 5

$\underline{b} - \underline{a} = (2\underline{i} - 5\underline{j}) - (2\underline{i} + \frac{4}{5}\underline{j})$

$= -\frac{29}{5}\underline{j}$ 5

$\underline{a} \cdot (\underline{b} - \underline{a}) = |a||\underline{b} - \underline{a}| \cos\theta$

$(2\underline{i} + \frac{4}{5}\underline{j})(-\frac{29}{5}\underline{j}) = \frac{\sqrt{116}}{5} \times \frac{29}{5} \cos\theta$ 5

$-\frac{4 \times 29}{5 \times 5} = \frac{\sqrt{116} \times 29}{5 \times 5} \cos\theta$

$\cos\theta = \frac{-4}{\sqrt{116}} \Rightarrow \theta = \cos^{-1}\left(\frac{-4}{\sqrt{116}}\right)$ 5

03. $\rightarrow s = ut + \frac{1}{2}at^2 \Rightarrow 40 = 30\cos\theta\, t \Rightarrow t = \frac{4}{3\cos\theta}$ 5

$\uparrow s = ut + \frac{1}{2}at^2 \Rightarrow 10 = 30\sin\theta \frac{4}{3\cos\theta} - \frac{1}{2} g \cdot \frac{16}{9\cos^2\theta}$ 5

$1 = 4\tan\theta - \frac{8}{9}(1 + \tan^2\theta)$

$8\tan^2\theta - 36\tan\theta + 17 = 0$ $\tan\alpha$, $\tan\beta$

මෙහි $\Delta > 0$ ∴ ප්‍රභින්න ද 2ක් පවතී 5

$\alpha + \beta = \frac{36}{8}$, $\alpha\beta = 17/8$ } $8x^2 - 36x + 17 = 0$ මූල α හා β වේ.

$\tan\alpha + \tan\beta = \frac{36}{8}$ 5

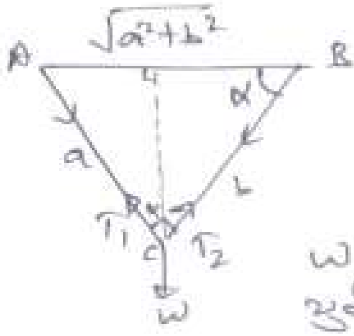
$\tan\alpha \tan\beta = \frac{17}{8}$

$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{36/8}{1 - 17/8}$

$\tan(\alpha + \beta) = -4$ 5

(04).



$AC^2 + BC^2 = AB^2$ වන බැවින්
$A\hat{C}B = 90^\circ$ වේ (5)

C හි සමතුලිතතාවය සලකා ලාමිගේ ප්‍රමේයය යෙදීමෙන්

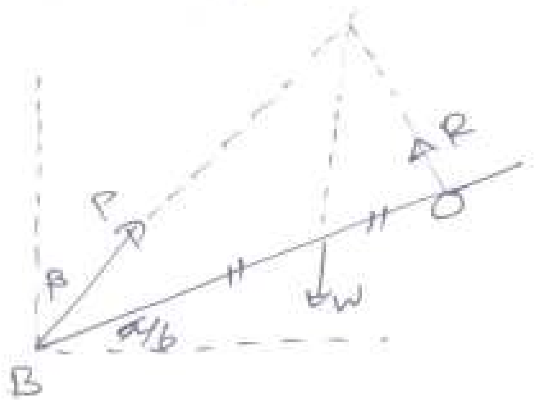
$$\frac{T_1}{\sin(90+\alpha)} = \frac{W}{\sin 90} = \frac{T_2}{\sin(180-\alpha)} \quad (10)$$

$$\frac{T_1}{\cos\alpha} = W = \frac{T_2}{\sin\alpha}$$

$$T_1 = W\cos\alpha \qquad T_2 = W\sin\alpha$$

$$= \frac{Wb}{\sqrt{a^2+b^2}} \quad (5) \qquad = \frac{Wa}{\sqrt{a^2+b^2}} \quad (5)$$

(05)



B ට $Wa\cos\pi/6 = R(2a)$

$$R = \frac{\sqrt{3}W}{4} \quad (5)$$

$\uparrow$ $P\cos\beta + R\cos\frac{\pi}{6} = W$ (5)

$$P\cos\beta = W - \frac{\sqrt{3}}{4}\cdot\frac{\sqrt{3}W}{2} = \frac{5W}{8} \quad (5)$$

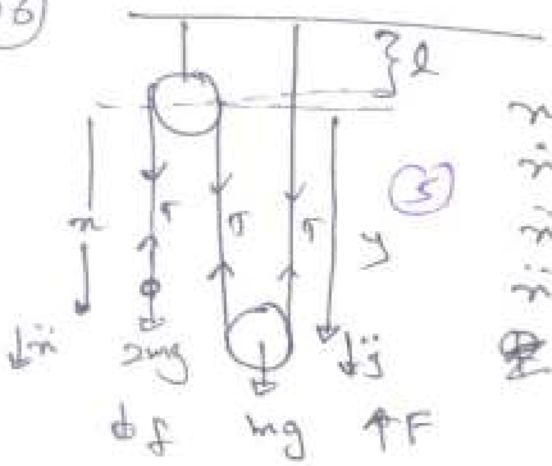
$\rightarrow$ $P\sin\beta = R\sin\pi/6 = \frac{\sqrt{3}W}{4}\cdot\frac{1}{2}$

$$= \frac{\sqrt{3}W}{8} \quad (5)$$

$$\tan\beta = \frac{P\sin\beta}{P\cos\beta} = \frac{\sqrt{3}W/8}{5W/8} \quad (5)$$

$$\tan\beta = \frac{\sqrt{3}}{5}$$

(06)



$x + 2y + l = k$

$\dot{x} + 2\dot{y} = 0$ (5)

$\ddot{x} + 2\ddot{y} = 0$

$\ddot{x} = -2\ddot{y}$ (5)

2m $\downarrow$ $F = ma$

$2mg - T = 2m\ddot{x}$

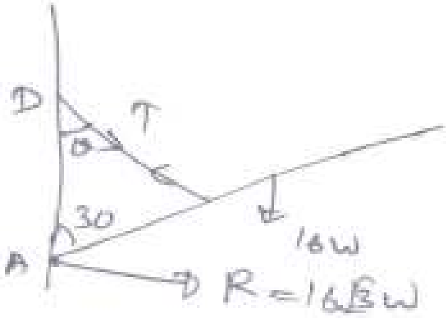
$2mg - T = 2mf$ (5)

m $\uparrow$ $F = ma$

$2T - mg = m(-\ddot{y})$ (5)

$2T - mg = mF$

07



$\uparrow \quad T\cos\theta = 16W \quad - (1)$ (5)

$\leftarrow \quad T\sin\theta = 16\sqrt{3}W \quad - (2)$ (5)

$\frac{(2)}{(1)} \quad \tan\theta = \sqrt{3}$

$\theta = \pi/3$ (5)

$A\hat{C}D = \pi/2$

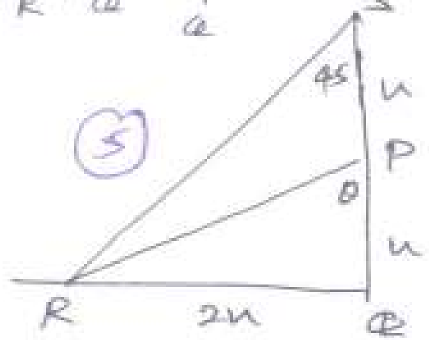
$AC = \sqrt{3}a \cos 30^\circ = \sqrt{3}a \cdot \frac{\sqrt{3}}{2} = \frac{3a}{2}$ (10)

08. ඔරුව - A ජලය - W පොළව - E

$V_{AE} = u\uparrow \quad V_{W,A} = \leftarrow \quad | \quad V_{AE} = \uparrow u \quad V_{WA} = \swarrow$

$V_{W,E} = V_{W,A} + V_{A,E}$ (5) $\qquad V_{WE} = V_{W,A} + V_{AE}$ (5)

$PK = {}_K\overleftarrow{\ }_Q + \uparrow u \qquad PR = \swarrow + \uparrow u$



(5)

$QS = QR = 2u$

$PR = \sqrt{(2u)^2 + u^2} = \sqrt{5}u$ (5)

~~ජලය ගලන දිශාව~~

ජලය ගලන දිශාව සමග θ ක් ආනත වේ

$\tan\theta = \frac{2u}{u} = 2 \Rightarrow \theta = \tan^{-1}(2)$ (5)

09.

$\rightarrow \quad s = ut + \frac{1}{2}at^2 \Rightarrow k = u\cos 60 \cdot t_1 \quad - (1)$ (5)

$\uparrow \quad h = u\sin 60\, t_1 - \frac{1}{2}gt_1^2 \quad - (2)$ (5)

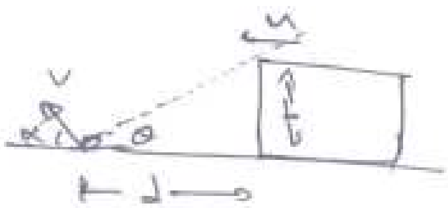
(1) න්, $t_1 = \frac{2k}{u}$

$h = u \cdot \frac{\sqrt{3}}{2} \times \frac{2k}{u} - \frac{1}{2}g \cdot \frac{4k^2}{u^2}$ (5)

$h = \sqrt{3}k - \frac{2gk^2}{u^2}$

$h + \frac{2gk^2}{u^2} = \sqrt{3}k$ (10)

(10) $V_{ME} = \overleftarrow{u}$ $\quad V_{BE} = \underset{\alpha}{\nwarrow}$



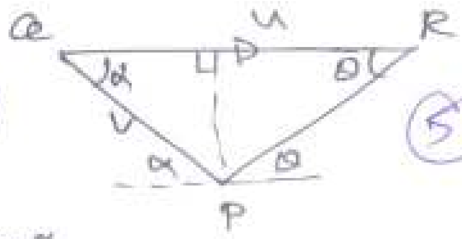
බෝලය බිත්තියේ කොනෙහි ස්පර්ශ
කිරීමට නම්

$V_{B,M} = \underset{\theta}{\nearrow}$ විය යුතුයි (5)

එවිට $\tan\theta = \frac{b}{d}$ වේ.

$V_{BM} = V_{BE} + V_{EM}$

$\underset{P\ \ \theta}{\nearrow} R = \underset{\alpha\ \ P}{\nwarrow} Q + \overrightarrow{u}$ (5)



(5)

$PS = V\sin\alpha \quad QS = V\cos\alpha$

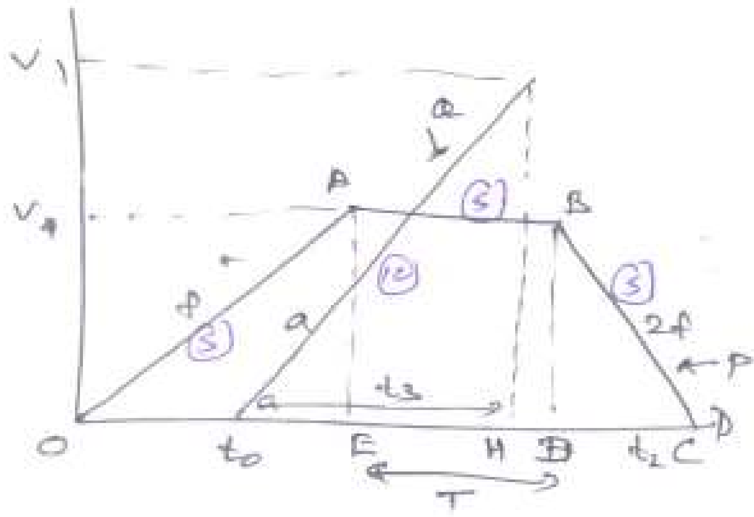
$SR = u - V\cos\alpha$ ($V < u$ නිසා, ඒනිසා $V\cos\alpha < u$ නිසා)

$\tan\theta = \frac{PS}{SR}$ (5)

$\frac{b}{d} = \frac{V\sin\alpha}{u - V\cos\alpha} \Rightarrow$ $bu - bV\cos\alpha = dV\sin\alpha$

$bu = V(d\sin\alpha + b\cos\alpha)$ (5)

(11)



OAE Δ යෙන්,

$f = \frac{V}{t_1}$

$t_1 = \frac{V}{f}$ (10)

BCD Δ යෙන්,

$2f = \frac{V}{t_2}$

$t_2 = \frac{V}{2f}$ (10)

OABC ව.ඵ $= d$

$\frac{1}{2} \times \left(T + T + \frac{V}{f} + \frac{V}{2f}\right) V = d$ (15)

$2T + \frac{3V}{2f} = \frac{2d}{V}$

$T = \frac{d}{V} - \frac{3V}{4f}$ (10)

FGH Δ යෙන්,

$a = \frac{V_1}{t_3}$

$t_3 = \frac{V_1}{a}$ (10)

FGH ව.ඵ $= D$

$\frac{1}{2} \times t_3 \times V_1 = D$ (10)

$\frac{1}{2} \times \frac{V_1}{a} \times V_1 = D$

$V_1^2 = 2aD$

$V_1 = \sqrt{2aD}$ (5)

$t_3 = \sqrt{\frac{2D}{a}}$ (10)

$t_0 + t_3$ කාලයේදී P ගිය දුර $= D$

$D = \frac{1}{2} \times t_1 \times V + V\left(t_3 - (t_1 - t_0)\right)$ (15)

$= \frac{1}{2} \cdot \frac{V^2}{f} + V\sqrt{\frac{2D}{a}} - \frac{V^2}{f} + Vt_0$ (15)

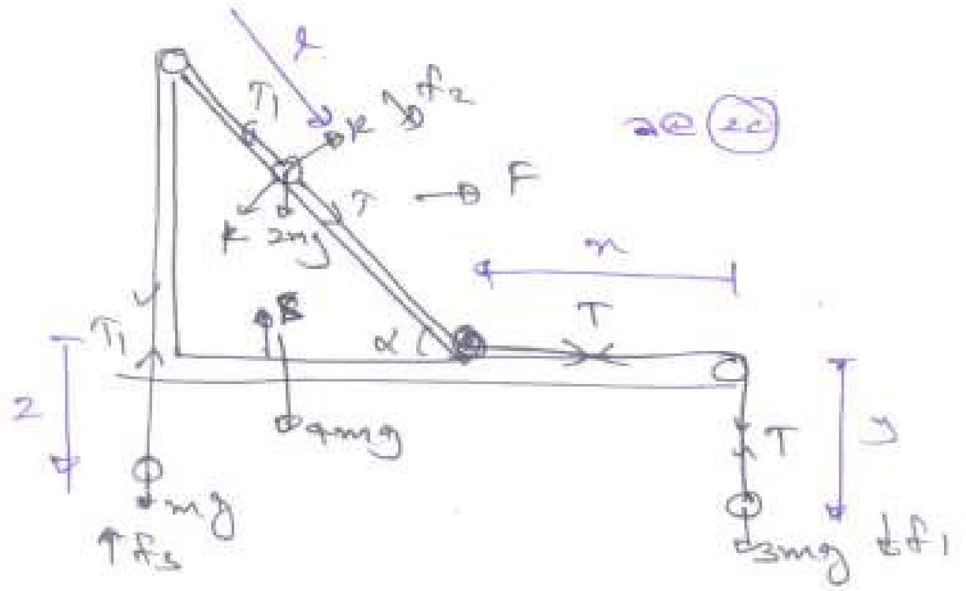
$= V\sqrt{\frac{2D}{a}} - \frac{V^2}{2f} + Vt_0$

$t_0 = \frac{D}{V} + \frac{V}{2f} - \sqrt{\frac{2D}{a}}$ (25)

150

(12)

b



බල (20)

$l + z + a = k$ (5)

$\ddot{l} + \ddot{z} = 0$

$f_2 - f_3 = 0$

$f_2 = f_3$

$x + y + b - l = k'$

$\ddot{x} + \ddot{y} - \ddot{l} = 0$ (5)

$-F + f_1 - f_2 = 0$

$F = f_1 - f_2$

$\underset{\sim}{a}_{m,E} = \uparrow f_3 \; \rightarrow F$ ත්වරණ (20)

$\underset{\sim}{a}_{2m,4m} = f_2$ (at θ)

$\underset{\sim}{a}_{3m,E} = \downarrow f_1$

$\underset{\sim}{a}_{4m,E} = \xrightarrow{\;F\;}$

$a_{2m,E} = \rightarrow F, \; f_2$ (at α)

$m \downarrow F = ma$

$mg - T_1 = -m f_3$ — (1) (15)

$2m \searrow F = ma$

$2mg\sin\alpha + T - T_1 = 2m(f_2 - F\cos\alpha)$ (20)

$3m \downarrow F = ma$

$3mg - T = 3mf_1$ (15)

$4m + 2m + m \xrightarrow{} F = ma$

$T = 4mF + 2m(F + f_2\cos\alpha) + mF$ (25)

දට ළඟා වීමට ගතවන කාලය t_1 නම්.

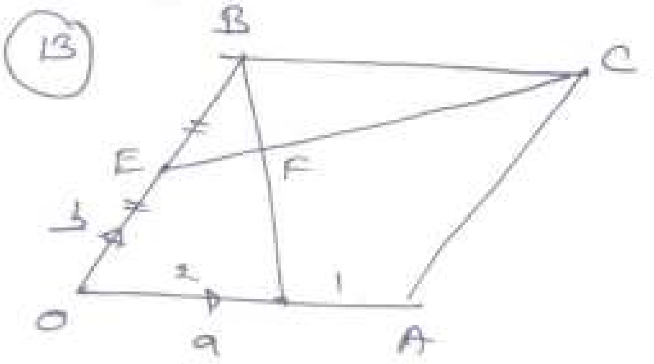
$s = ut + \frac{1}{2}at^2$

$\frac{c}{2} = \frac{1}{2} g\sin\alpha \; t_1^2$ (15)

$t_1^2 = \frac{c}{g\sin\alpha}$ (10)

150

13



$\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OD}$ 10

$= -\underline{b} + \frac{2}{3}\overrightarrow{OA}$ 10

$= -\underline{b} + \frac{2}{3}\underline{a}$

$= \frac{1}{3}(2\underline{a} - 3\underline{b})$ 10

$\overline{CE} = \overline{CB} + \overline{BE}$

$= -a + \frac{1}{2}\overline{BO}$ 10

$= -\underline{a} \cancel{+} \frac{1}{2}\underline{b}$

$= \frac{1}{2}(\underline{b} + 2a)$ 10

$\overrightarrow{FE} = \lambda\overline{CE} \qquad \overline{BF} = \mu\overline{BD}$

$\overline{BE} = \overline{BF} + \overline{FE}$

$-\frac{1}{2}\underline{b} = \mu\overline{BD} + \lambda\overline{CE}$ 10

$-\frac{1}{2}\underline{b} = \mu\left(\frac{1}{3}(2\underline{a} - 3\underline{b})\right) - \frac{\lambda}{2}(\underline{b} + 2\underline{a})$ 10

$-\frac{1}{2}\underline{b} = \underline{a}\left(\frac{2}{3}\mu - \lambda\right) + \underline{b}\left(-\mu - \frac{\lambda}{2}\right)$

$\left(\frac{2}{3}\mu - \lambda\right)\underline{a} + \underline{b}\left(\mu + \frac{\lambda}{2} - \frac{1}{2}\right) = 0$

$\frac{2}{3}\mu - \lambda = 0$ 5 ෆ $\mu + \frac{\lambda}{3} = \frac{1}{2}$ 5 $\Rightarrow \lambda = \frac{1}{4}$ 10 $\mu = \frac{3}{8}$ 10

$\overline{BD} \perp \overline{CE}$ නම්

$\overline{BD} \cdot \overline{CE} = 0$ 10

$\frac{1}{3}(2\underline{a} - 3\underline{b}) \cdot \left[\frac{1}{2}(\underline{b} + 2\underline{a})\right] = 0$ 10

$2\underline{a}\cdot\underline{b} + 4\underline{a}\cdot\underline{a} - 3\underline{b}\cdot\underline{b} - 6\underline{a}\cdot\underline{b} = 0$

$4|\underline{a}|^2 - 4\underline{a}\cdot\underline{b} - 3|\underline{b}|^2 = 0$ 10

OACB රොම්බසයක් නම් $OA = OB \Rightarrow |a| = |b|$

$1 = 4\cos\theta$

$\cos\theta = \frac{1}{4}$ 10

$\theta = \cos^{-1}\left(\frac{1}{4}\right)$

OACB සෘජුකෝණාස්‍රයකි.

$\underline{a}\cdot\underline{b} = 0$

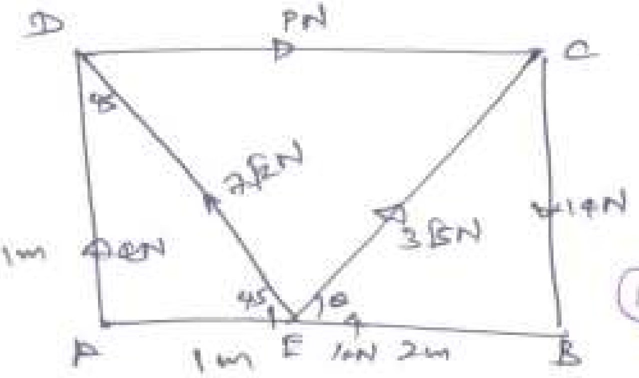
$4|a|^2 - 3|\underline{b}|^2 = 0$

$|a|^2 = \frac{3|\underline{b}|^2}{4}$

$|a| = \frac{\sqrt{3}}{2}|\underline{b}|$ 10

150

(14)



$\tan\theta = \frac{1}{2}$

$\cos\theta = \frac{2}{\sqrt{5}}$

$\sin\theta = \frac{1}{\sqrt{5}}$ (5)

(10)

$\xrightarrow{} x = P - 10 - 7\sqrt{2}\cdot\frac{1}{\sqrt{2}} + 3\sqrt{5}\cos\theta$ (10)

$= P - 10 - 7 + 3\sqrt{5}\cdot\frac{2}{\sqrt{5}}$

$x = P - 11$ (5)

$\uparrow Y = Q - 14 + 7\sqrt{2}\cdot\frac{1}{\sqrt{2}} + 3\sqrt{5}\sin\theta$ (10)

$= Q - 14 - 7 + 3\sqrt{5}\cdot\frac{1}{\sqrt{5}}$

$Y = Q - 14$ (5)

D↺ $G = -10\times1 - 14\times3 + 3\sqrt{5}\sin\theta\times3$ (20)

$= -10 - 42 + 3\sqrt{5}\cdot\frac{1}{\sqrt{5}}\cdot3 = -43$

$G↻ = 43$ Nm (5)

$R \neq 0$ හා $G \neq 0$ නිසා පද්ධතිය සමතුලිත නොවේ.

පද්ධතිය බල යුග්මයකට ඌනනය වේ නම් $R = 0$ හා $G \neq 0$.

$X = 0 \Rightarrow P = 11$ (10) හා $Y = 0 \Rightarrow Q = 4$ (10)

II $x = 7 - 11 = -4$N (5) $y = 8 - 4 = 4$N (5)

$R = \sqrt{16+16} = 4\sqrt{2}$ N (10) $\tan\alpha = \frac{4}{4} = 1$

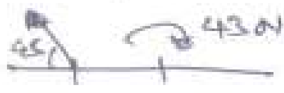
$\alpha = \pi/4$ (10)

සම්ප්‍රයුක්තය ED ට සමාන්තර වන අතර E සිට D දිශාවට ක්‍රියා කරයි.

D වටා සම්ප්‍රයුක්තයේ ඝූර්ණය = බල පද්ධතියේ ඝූර්ණය

$43 = 4x$

$x = 43/4$ m (10)



$+ M =$

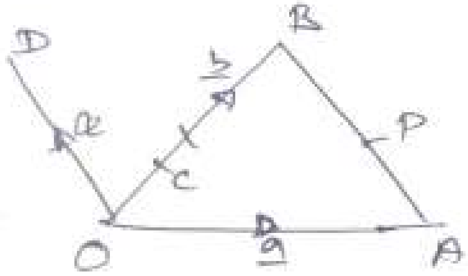
(10) (150)

D↺ $43 + M = -4\times3$ (5)

$M = -12 - 43 = -55$ Nm $\Rightarrow M = 55$ Nm

දක්ෂිණාවර්ත වේ. (5)

15. I



$\overline{AC} = \overline{AO} + \overline{OC}$ (10)

$= -\overline{OA} + \frac{1}{3}\overline{OB}$ (5)

$= -\underline{a} + \frac{1}{3}\underline{b}$ (5)

$\overline{AD} = \overline{AO} + \overline{OD}$ (5)

$= -\overline{OA} + \frac{1}{2}\overline{AB}$ (10)

$= -a + \frac{1}{2}(AO + OB)$

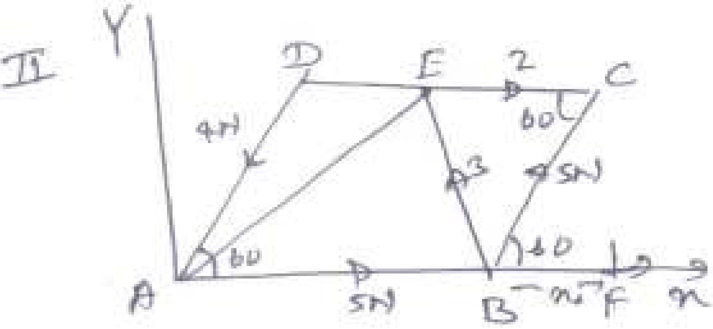
$= -a + \frac{1}{2}(-a + b)$

$= -\frac{3}{2}a + \frac{1}{2}\underline{b}$ (10)

$\overline{AD} = +\frac{3}{2}(-a + \frac{1}{3}\underline{b})$ (10)

$\overline{AD} = +\frac{3}{2} AC$ (10)

II



$\overrightarrow{x} = 5 + 5\cos 60 - 3\cos 60 - 4\cos 60 + 2$ (10)

$= 7 - 2\cos 60$

$= 6$ (5)

$\uparrow Y = 5\cos 30 + 3\cos 30 - 4\cos 30$ (10)

$= 4\cos 30$

$Y = 2\sqrt{3}$ (5)

$\tan\theta = \frac{Y}{X} = \frac{2\sqrt{3}}{6} = \frac{1}{\sqrt{3}}$ (10)

$\theta = \pi/6$ ∴ සම්ප්‍රයුක්තය AE ඔස්සේ වේ. (5)

$R = \sqrt{x^2 + y^2} = \sqrt{36 + 12} = 4\sqrt{3}\ N$ (10)

සම්ප්‍රයුක්තයේ රේඛාව, දික්කළ AB හමුවන F හිදී ($CBF = x_0$) යැයි ගනිමු. B වටා ඝූර්ණ ගැනීමෙන්

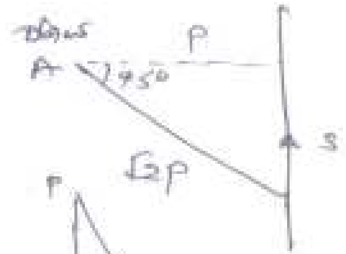
B ↻ $4 \times 2\sin 60 - 2 \times 1\sin 60 = 2\sqrt{3} \times x_0$ (20)

$8\frac{\sqrt{3}}{2} - 2\frac{\sqrt{3}}{2} = 2\sqrt{3}\, x_0$

$x_0 = 3/2\ m$ // (10)

150

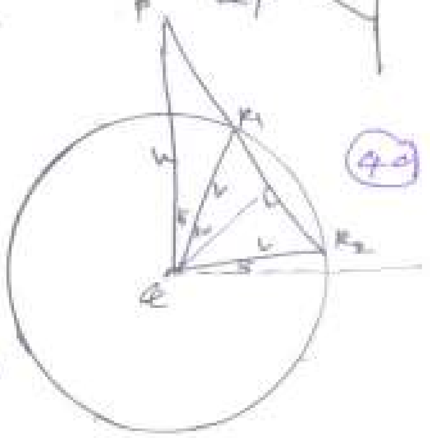
16. නැව S   බෝට්ටු B_1, B_2



$V_{SE} = \uparrow u$ (5) $V_{B_iE} = V$ (5)

$V_{B_iS} = V_{B_iE} + V_{ES}$ (10)

$\overrightarrow{45°} = V + \downarrow u$



(40)

$QS = u\sin 45 = v\sin\theta$ (10)

$v\sin\theta = \frac{u}{\sqrt{2}} \Rightarrow \frac{u}{v} = \sqrt{2}\sin\theta$

$\frac{v}{u} = \frac{\sqrt{2}}{\sqrt{3}}$ බව දී ඇත. $\cancel{\frac{\sqrt{2}}{\sqrt{3}} = \sqrt{2}\sin\theta}$

$\sqrt{\frac{3}{2}} = \sqrt{2}\sin\theta$ (10)   (10)

$\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \pi/3$ (5)

විට $P\hat{Q}R_1 = 60° - 45° = 15°$ ... 15° ... (5)

15° ...

$T_1 = \frac{\sqrt{2}P}{PR_2}$ (5) $T_2 = \frac{\sqrt{2}P}{PR_2}$ (5)

$T_2 - T = \frac{\sqrt{2}P}{PR_1} - \frac{\sqrt{2}P}{PR_2} = \sqrt{2}P\left[\frac{1}{PR_1} - \frac{1}{PR_2}\right]$ (10)

$= \sqrt{2}P\left[\frac{PR_2 - PR_1}{PR_1 \cdot PR_2}\right] = \sqrt{2}P\left[\frac{(PS+SR_2)-(PS-SR_1)}{(PS+SR_2)(PS-SR_1)}\right]$

$= \sqrt{2}P\left[\frac{SR_2 + SR_2}{PS^2 - SR_2^2}\right]$ $SR_1 = SR_2$ නිසා

$= \frac{2\sqrt{2}P\,SR_2}{PS^2 - SR_2^2} = \frac{2\sqrt{2}P \cdot \frac{v}{2}}{\left(\frac{v}{\sqrt{2}}\right)^2 - \left(\frac{v}{2}\right)^2}$ (10)

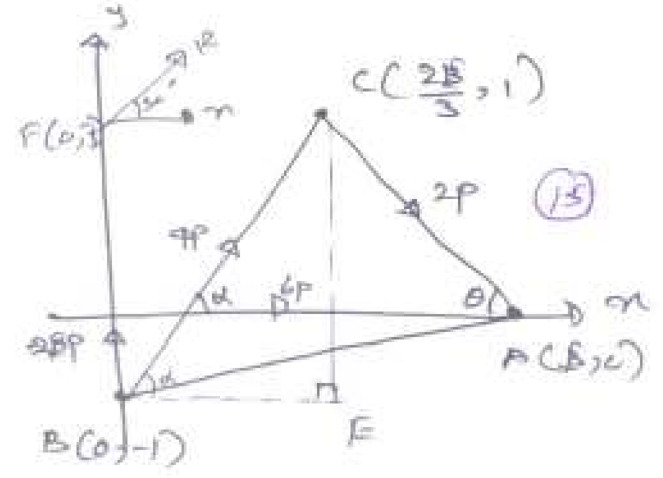
$= \frac{\sqrt{2}Pv}{\frac{v^2}{2} - \frac{v^2}{4}} = \frac{\sqrt{2}P \cdot \frac{u\sqrt{2}}{\sqrt{3}}}{\frac{u^2}{2} - \frac{2u^2}{3} \times \frac{1}{4}} = \frac{\frac{2Pu}{\sqrt{3}}}{u^2\left[\frac{1}{2} - \frac{1}{6}\right]}$ (10)

$= \frac{2P}{\sqrt{3}u} \times \frac{1}{1/3} = \frac{2P}{\sqrt{3}u} \times 3 = \cancel{\frac{2\sqrt{3}P}{3}}$

$= \frac{2\sqrt{3}P}{u}$ (10)

(150)

17.



(15)

$\tan\theta = \dfrac{1}{\sqrt{3} - \frac{2\sqrt{3}}{3}} = \dfrac{1}{\frac{\sqrt{3}}{3}} = \sqrt{3}$

$\theta = \pi/3$ (10)

BCE Δ න්,

$\tan\alpha = \dfrac{1-(-1)}{\frac{2\sqrt{3}}{3}} = \sqrt{3}$

$\alpha = \pi/3$ (10)

$\rightarrow \; x = 2P\cos 60 + 4P\sin 60 + 6P$ (10)

$\quad = 9P$ (5)

$\uparrow \; Y = 4P\sin 60 - 2P\sin 60 + 2\sqrt{3}P$ (10)

$\quad = 3\sqrt{3}P$ (5)

$R = \sqrt{x^2+y^2} = \sqrt{18P^2+27P^2} = 6\sqrt{3}P$ (10)

$\tan\beta = \dfrac{Y}{X} = \dfrac{3\sqrt{3}P}{9P} = \dfrac{1}{\sqrt{3}}$ (10)

$\beta = \pi/6$

සම්ප්‍රයුක්ත බලය F හිදී y අක්ෂය කපයි නම් C වටා ඝූර්ණ ගැනීමෙන් ; $F \equiv (0, \bar{y})$ ලෙස ගත් විට

C↺ $\; -6P\times 1 + 2\sqrt{3}P \times \frac{2\sqrt{3}}{3} = 3\sqrt{3}P \times \frac{2\sqrt{3}}{9} + 9P\times(\bar{y}-1)$ (20)

$-6+4 = 6 + 9\bar{y} - 9$

$1 = 9\bar{y}$

$\bar{y} = 1/9 \Rightarrow F \equiv \left(0, \frac{1}{9}\right)$ (10)

සම්ප්‍රයුක්ත බලයේ ක්‍රියා රේඛාවේ සමීකරණය

$y - \frac{1}{9} = \frac{1}{\sqrt{3}}(x-0) \Rightarrow 9y - 1 = 3\sqrt{3}x$ (15)

AB දිගේ $6\sqrt{3}P$ බලය යෙදීම ($O\hat{A}B = 30°$ නිසා) සම්ප්‍රයුක්තය එක යුග්මයක් වේ

එම යුග්මයේ ඝූර්ණය $= (6\sqrt{3}P)\times\left(1+\frac{1}{9}\right)\sin 60$ (10)

$= 6\sqrt{3}P \times \frac{10}{9} \times \frac{\sqrt{3}}{2}$

$= 10P$ (10)

150

සංශෝධන

(3) ප්‍රශ්නය $g = 10\ ms^{-2}$ ලෙස ගන්න.