"නැණ සයුර" අධ්‍යාපනික වැඩසටහන - 2023

සරසවි පිවිසුම් ඇන්වැල

උතුරුමැද පළාත් අධ්‍යාපන දෙපාර්තමේන්තුව

සංයුක්ත ගණිතය - I පත්‍රය

උත්තර පත්‍රය.

12 ශ්‍රේණිය

කාලය : පැය 03 මිනිත්තු 10

නම :..

උපදෙස් :

- මෙම ප්‍රශ්න පත්‍රය කොටස් දෙකකින් සමන්විත වේ.
- *A* කොටස (ප්‍රශ්න 1-10) සහ *B* කොටස (ප්‍රශ්න 11-17)
- *A* කොටස

  සියලුම ප්‍රශ්නවලට පිළිතුරු සපයන්න. එක් එක් ප්‍රශ්නය සඳහා ඔබේ පිළිතුරු සපයා ඇති ඉඩෙහි ලියන්න. වැඩිපුර ඉඩ අවශ්‍ය වේ නම් ඔබට අමතර ලියන කඩදාසි භාවිත කළහැකිය.
- *B* කොටස

  ප්‍රශ්න පහකට පමණක් පිළිතුරු සපයන්න.

පරීක්ෂකවරුන්ගේ ප්‍රයෝජන සඳහා පමණි.

(10) සංයුක්ත ගණිතය I		
කොටස	ප්‍රශ්න අංක	ලකුණු
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
එකතුව		

I පත්‍රය	
II පත්‍රය	
එකතුව	
අවසාන ලකුණු	

අවසාන ලකුණු

ඉලක්කමෙන්	
අකුරින්	

12 ශ්‍රේණිය සංයුක්ත ගණිතය I
[illegible] ප්‍රශ්න

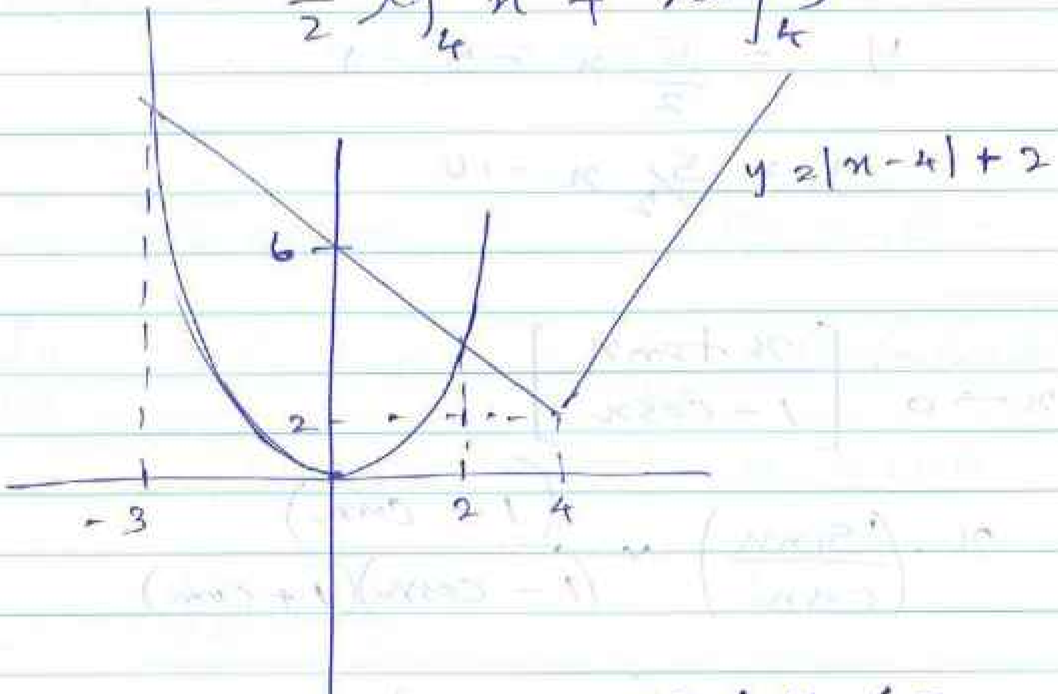
1/ $\log_{16} xy^2 = \frac{1}{2}\log_4 x + \log_4 y$

$$\log_{16} xy^2 = \frac{\log_4 (xy)^2}{\log_4 16}$$

$$= \frac{1}{2}\left(\log_4 x + \log_4 y^2\right)$$

$$= \frac{1}{2}\log_4 x + \log_4 y$$

2/



$+3 < x < 2$

3/ $$\frac{3x+5}{(3x-1)(3x+2)} = \frac{A}{3x-1} + \frac{B}{3x+2}$$

$$(3x+5) = A(3x+2) + B(3x-1)$$

x/ $3 = 3A + 3B$ —— ①

නිය $5 = 2A + B$ —— ②

① හා ② න් $3A = 6 \quad A = 2 \quad b = -1$

4/ $2x + 5y - 4 = 0$

$m = \frac{-2}{5}$

$m m_1 = -1$

$\left(\frac{-2}{5}\right) m_1 = -1$

$m_1 = 5/2$

$(2, -5)$

$[y - (-5)] = 5/2 (x - 2)$

$y = \frac{5}{2} x - 5 - 5$

$y = 5/2\, x - 10$

5/ $x \to 0 \left[\frac{x \tan x}{1 - \cos x}\right]$

$= x \cdot \left(\frac{\sin x}{\cos x}\right) \times \frac{(1 + \cos x)}{(1 - \cos x)(1 + \cos x)}$

$= x \times \frac{\sin x}{\cos x} \times \frac{(1 + \cos x)}{1 - \cos^2 x}$

$= \underset{x \to 0}{x} \times \frac{\sin x}{\cos x} \times \frac{(1 + \cos x)}{\sin^2 x}$

$= \underset{x \to 0}{\frac{x}{\sin x}} \times \underset{x \to 0}{\frac{1 + \cos x}{\cos x}}$

$= 1 \times \frac{1 + 1}{1}$

$= \underline{\underline{2}}$

6/ $f(x) = 5 - x^3$ $\qquad x = \pm 5$

$g(x) = \frac{1}{25 - x^2}$

$g o f(x) = \frac{1}{25 - (5 - x^3)^2}$

$= \frac{1}{\cancel{-4 + x^3}} \qquad \frac{1}{25 - (25 - 10x^3 + x^6)}$

$g o f(x) = \frac{1}{x^3(10 + x^3)}$

7/ $x = 2\cos\theta \qquad y = \sqrt{3}\sin\theta$

$\frac{dx}{d\theta} = 2(-\sin\theta) \qquad \frac{dy}{d\theta} = \sqrt{3}(+\cos\theta)$

$= -2\sin\theta \qquad = \sqrt{3}\cos\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$

$= \frac{\sqrt{3}\cos\theta}{-2\sin\theta}$

$= \frac{-\sqrt{3} \times \cancel{3}\,^1/_2}{-2 \times \sqrt{3}/2} = -\frac{1}{2}$

$\theta = \pi/3 \Rightarrow (1, 3/2)$

$y - 3/2 = -1/2(x - 1)$

$2y + x - 4 = 0$

08/ $(1, 2)\ (2, 9)$

$y - 9 = \frac{7}{1}(x - 2)$

$y - 7x + 5 = 0$ —— ①

$3x - y - 9 = 0$ —— ②

[illegible] $(-1, -12)$

$\sqrt{441 + 9} : \sqrt{196 + 4}$

$\sqrt{450} : \sqrt{200}$

$\sqrt{9} : \sqrt{4}$

$3 : 2$

9/ $2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$2\sin x = -1$ $\sin x = 1$

$\sin x = -1/2$ $\sin x = 1$

$\sin x = \sin(-\pi/6)$ $\sin x = \sin\pi/2$

$x = n\pi + (-1)^n(-\pi/6)$ ~~$x = 90\pi/2$~~

$x = n\pi + (-1)^n \pi/2$

$x = 7\pi/6,\ \frac{11\pi}{6}$ $= \pi/2,$

10/ $(x - 3)(x + 2) = \alpha(x - 1)$

$x^2 + x - 6 = \alpha x - \alpha$

$x^2 - x(1 + \alpha) + \alpha - 6 = 0$

$\Delta = (1 + \alpha)^2 - 4.(\alpha - 6)$

$= \alpha^2 + 2\alpha + 1 - 4\alpha + 24$

$= \alpha^2 - 2\alpha + 25$

$(\alpha - 1)^2 + 26 > 0$

11/ $ax^2 + bx + c = 0$

$\alpha\beta = c/a$ ⑤ $\alpha+\beta = -b/a$ ⑤

$\Delta = b^2 - 4ac$

$= a^2\left(\frac{b^2}{a^2} - \frac{4ac}{a^2}\right)$ ⑩

$= a^2\left(\cancel{4(\alpha+\beta)} \; \frac{b^2}{a^2} - 4 \cdot c/a \right)$ ⑤

$\alpha\beta < 0$ නම් $c/a < 0$ ⑤

$\therefore -4\, c/a > 0$ ⑤

$\therefore \Delta > 0$ ⑤

40/40

$(k-2)x^2 - 2(k-1)x + k = 0$

$\alpha\beta = \frac{k}{k-2}$ ⑩

$= \frac{k}{k-2} < 0$ ⑩ $0 < k < 2$

$\therefore$ මූල තාත්වික වේ ⑤

25/25

$(k-2)x^2 - 2(k-1)x + k = 0$

$\alpha+\beta = \frac{2(k-1)}{k-2}$ $\alpha\beta = \frac{k}{k-2}$

⑤ ⑤

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ⑤

$= \left[\frac{2(k+1)}{k-2}\right]^2 - 2 \cdot \frac{k}{k-2}$ ⑤

$= \frac{4(k^2+2k+1)}{(k-2)^2} - \frac{2k(k-2)}{(k-2)^2}$

$= \frac{2k^2-4k+4}{(k-2)^2}$

$= \frac{2(k^2-2k+2)}{(k-2)^2}$ ⑩

$\alpha^2\beta^2 = \frac{k^2}{(k-2)^2}$ ⑤

$x^2 - \frac{2(k^2-2k+2)x}{(k-2)^2} + \frac{k^2}{(k-2)^2} = 0$ ⑩

$(k-2)^2 - 2(k^2-2k+2)x + k^2 = 0$

45/45

b/ $64^{\frac{1}{x}} - 2^{\frac{3x+3}{x}} + 12 = 0$

$(2^6)^{1/x} - 2^{3+3/x} + 12 = 0$

$(2^{3/x})^2 - 8 \times 2^{3/x} + 12 = 0$ ⑩

$2^{3/x} = y$

$y^2 - 8y + 12 = 0$

$(y-6)(y-2) = 0$ ⑩

$y - 6 = 0$

$y = 6,$

$2^{3/x} = 6$ ⑤

$\log_2 6 = \frac{3}{x}$

$x = \frac{3}{\log_2 6}$ ✓ ⑤

$y - 2 = 0$

$y = 2$

$2^{3/x} = 2$ ⑤

$3/x = 1$ ⑤

$x = 3$ ✓

12 $f(x) = ax^2 + 2x + 2b \qquad g(x) = cx^2 + 2x + b$

$f(-1) = -6 \qquad f(2) = 12$

$a - 2 + 2b = -6 \qquad 4a + 4 + 2b = 12$

$a + 2b = -4 \quad — ① \qquad 4a + 2b = 8 \quad — ②$

① ② ল

$3a = 12$

$a = 4$

$b = -4$

$f(x) = 4x^2 + 2x - 8 \qquad g(x) = cx^2 + 2x - \cancel{8}4$

$h(x) = f(x) + g(x)$

$= 4x^2 + 2x - 8 + cx^2 + 2x - \cancel{8}\,4$

$h(x) = (4 + c)x^2 + 4x - \cancel{16}\, -12$

$h(-2) = 0$

$(4 + c)\,4 - 8 \overset{-12}{\cancel{+16}} = 0$

$4 + c = \cancel{6}\;5$

$c = \cancel{2}\;1$

$g(x) = \cancel{2}x^2 + 2x - \cancel{8}4$

$3/g(x)/ = 3\cancel{4}/x^{\cancel{2}} + \cancel{6}x/ - \cancel{24}^{12}$

b/.

$$\frac{\log_3 8}{\log_9 10 \cdot \log_4 16} = \frac{\log_{10} 8 \times \log_{10} 9 \times \log_{10} 4}{\log_{10} 3 \times \log_{10} 10 \times \log_{10} 16}$$

$$= \frac{\log_{10} 2^3 \times \log_{10} 3^2 \times \log_{10} 4}{\log_{10} 3 \times 1 \times \log_{10} 4^2}$$

$$= 3\frac{\log_{10} 2 \times \cancel{2}\cancel{\log_{10}} 3 \times \cancel{\log_{10} 4}}{\cancel{\log_{10}} 3 \times \cancel{2}\cancel{\log_{10}} 4}$$

$$= 3\log_{10} 2$$

13/ $$f(x) = \frac{(x-1)(x-5)}{x-3}$$

$$-3 \leq f(x) \leq 3$$

$$-3 \leq \frac{(x-1)(x-5)}{(x-3)}$$

$$\frac{(x-1)(x-5)}{(x-3)} + 3 \geqslant 0$$

$$\frac{(x-1)(x-5) + 3(x-3)}{x-3} \geqslant 0$$

$$\frac{x^2 - 3x - 4}{x-3} \geqslant 0$$

$$\frac{(x-4)(x+1)}{x-3} \geqslant 0$$

$$\frac{- \times -}{-} \qquad \frac{- \times +}{-} \qquad \frac{- \times +}{+} \qquad \frac{+ \times +}{+}$$

$$(-) \quad -1 \quad (+) \quad 3 \quad (-) \quad 4 \quad (+)$$

$-1 \leq x \ 3$, $x \geqslant 4$ —— Ⓐ

50

$$\frac{(x-1)(x-5)}{(x-3)} \leq 3$$

$$\frac{(x-1)(x-5)}{(x-3)} - 3 \leq 0$$

$$\frac{x^2 - 9x + 14}{(x-3)} \leq 0$$

$$\frac{(x-7)(x-2)}{x-3} \le 0$$

$\frac{-\times-}{-}$ $\quad$ $\frac{-\times+}{-}$ $\quad$ $\frac{-\times+}{+}$ $\quad$ $\frac{+\times+}{+}$

(−) $\quad$ 2 $\quad$ (+) $\quad$ 3 $\quad$ (−) $\quad$ 7 $\quad$ +

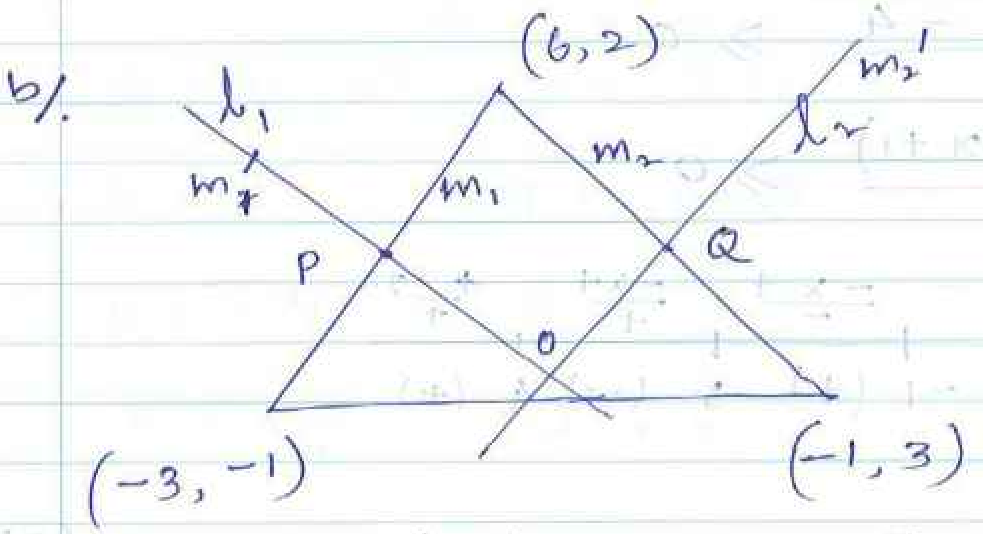
$x \le 2 \qquad 3 < x \le 7$ ——— Ⓑ

Ⓐ හා Ⓑ පොදු සත්‍යතා කුලකය

$-1 \le x \le 2$ හෝ $4 \le x \le 7$ වේ

40

b/.



$P \equiv \left(\frac{3}{2}, \frac{1}{2}\right) \qquad Q \equiv \left(\frac{5}{2}, \frac{5}{2}\right)$

$m_1 = \frac{3}{9} = \frac{1}{3} \qquad m_2 = \frac{1}{-7}$

$m_1' = -3 \qquad m_2' = 7$

30

$l_1 = y - \frac{1}{2} = -3\left(x - \frac{3}{2}\right)$

$2y - 1 = -6x + 9$

$6x + 2y = 10$ ——— ①

$l_2 = y - 5/2 = 7(x - 5/2)$

$2y - 5 = 14x - 35$

$14x - 2y = 30$ —— ②

① in ②nd

$20x = 40$

$x = 2$

$y = -1$

$O = (2, -1)$

$r^2 = (2+1)^2 + (6-2)^2$

$r = 5$

30

14/

b. $f(x) = \dfrac{1}{(x-1)^2(x+1)}$

$$f'(x) = \frac{-[(x-1)^{2}\,⑤ + (x+1)\,\overset{⑤}{2(x-1)}]}{(x-1)^4(x+1)^2\,⑤} \quad /$$

$$= \frac{-(x-1)[x-1+2x+2]}{(x-1)^4(x+1)^2}$$

$$f''(x) = \frac{-(3x+1)}{(x-1)^3(x+1)^2} \quad ⑩$$

$\frac{25}{25}$

$f'(x)=0$, $x = -1/3$ දී ස්ථාවර ලක්ෂ්‍ය ඇත

$x = 1/3$; $y = 27/32$

⑤

$x = 1$ සහ $x = -1$ දී සිරස් ස්පර්ශෝන්මුඛ

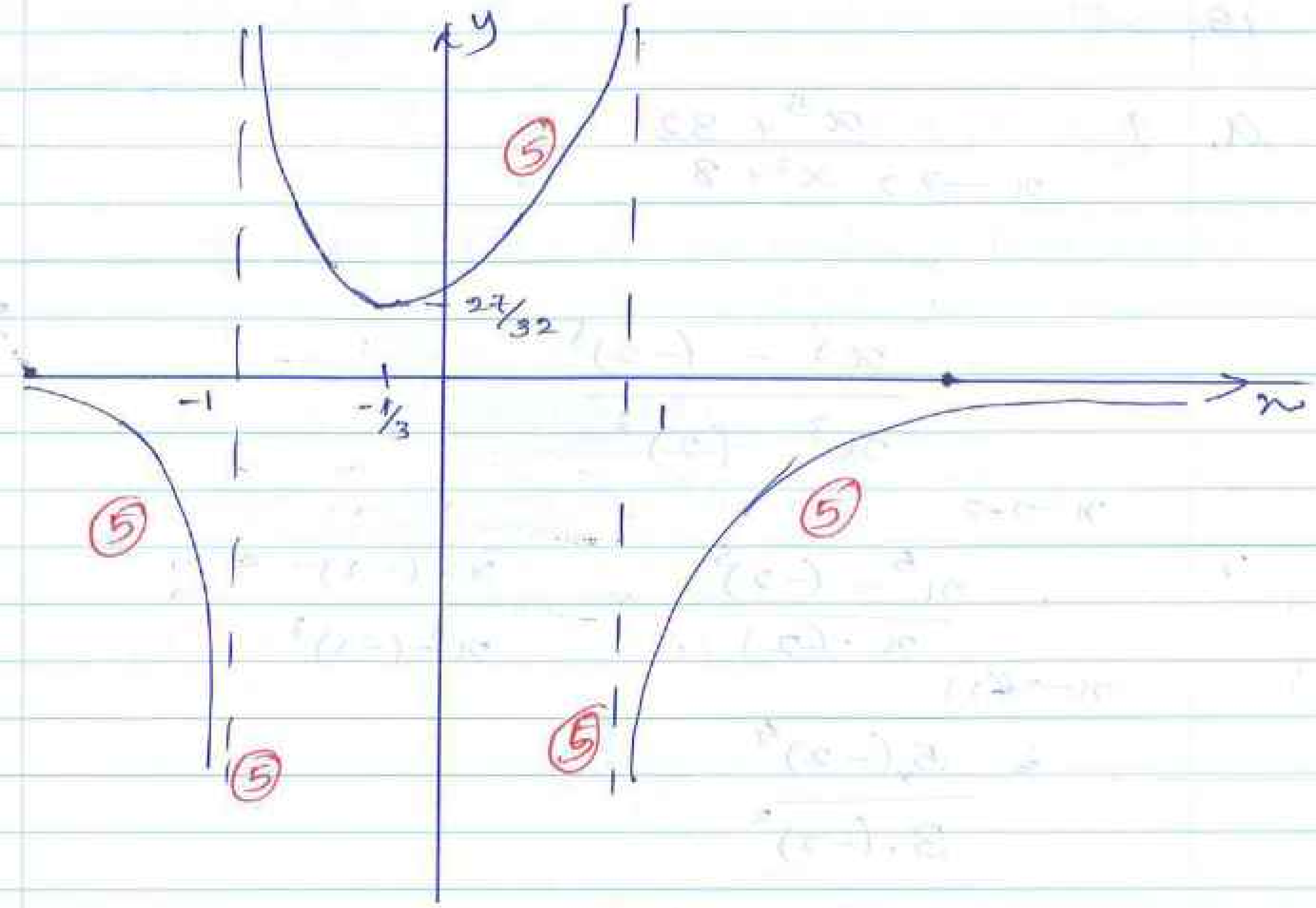
	$x < -1$	$-1 < x < -1/3$	$-1/3 < x < 1$	$x > 1$
$\frac{dy}{dx}$ හි ලකුණ	+	−	+	−
	↗	↘ ∪	↗	↖

⑳

⑩

$x \to +\infty \quad y \to 0$

$x \to -\infty \quad y \to 0$



a/ $y = (\sec x + \tan x)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(\sec x + \tan x)^{-1/2}\left[\sec x \tan x + \sec^2 x\right]$ ⑤ ⑤ & ⑤

$= \frac{\sec x}{2}(\sec x + \tan x)^{-1/2}(\sec x + \tan)$

$= \frac{\sec x}{2}(\sec x + \tan x)^{1/2}$ ⑩

$2\frac{dy}{dx} = y \sec x.$

$2\frac{d^2y}{dx^2} = y \sec x \tan x + \sec x \frac{dy}{dx}$ ~~⑤~~ ⑩ ~~⑤~~ ⑩

$= 2\frac{dy}{dx}\tan x + \sec x \frac{dy}{dx}$ ⑤

$2\frac{d^2y}{dx^2} = (\sec x + 2\tan x)\frac{dy}{dx}$ ⑩

15. ⑮

a. I $\frac{x^5+32}{x^3+8}$

$x \to 2$

=

$\frac{x^5-(-2)^5}{x^3-(-2)^3}$ ⑤

$x \to -2$

$= \frac{x^5-(-2)^5}{x-(-2)}$ ⑤ $\times \frac{x-(-2)}{x^3-(-2)^3}$ ⑤

$x \to (-2)$

$= \frac{5\times(-2)^4}{3\cdot(-2)^2}$

$= \frac{20}{3}$. ⑩

$\frac{25}{25}$

II $\frac{\tan^3 x - 3\tan x}{\cos(x\ \cancel{x} + \pi/6)}$

$x \to \pi/3$

$= \frac{\tan x(\tan^2 x - 3)}{\cos(x+\pi/6)}$ ⑤

$= \tan x \times \frac{\sin^2 x - 3\cos^2 x}{\cos^2 x \ \cos(x+\pi/6)}$

$x \to \pi/3$ $\quad$ $x \to \pi/3$

⑩

$= \sqrt{3}$ ⑤ $\times \frac{(\sin x + \sqrt{3}\cos x)(\sin x - \sqrt{3}\cos x)}{\cos^2 x.\ \cos(\cancel{x} + \pi/6)}$

$$= \frac{\sqrt{3}\left(\frac{\sqrt{3}}{2} + \sqrt{3} \times \frac{1}{2}\right)}{1/4} \quad ⑤ \quad \frac{2\left(\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x\right)}{\cos\left(x + \pi/6\right)} \quad ⑤$$

$$= 12 \times (-2) \frac{\left(\cos x \cos \pi/6 - \sin x \sin \pi/6\right)}{\cos\left(x + \pi/6\right)} \quad ⑤$$

$$= -24 \frac{\cos\left(x + \pi/6\right)}{\cos x + \pi/6} \quad ⑤$$

$$x \to \pi/3$$

⑤

$$= -24$$ //

45 / 45

b) $y = \sin(x+1)$ —— ①

x හි δx වෙනස්වීම සඳහා y හි වෙනස්වීම δy නම්

$y + \delta y = \sin(x + \delta x + 1)$ —— ②

⑤

② - ①

$$\frac{\delta y}{\delta x} = \frac{\sin(x + \delta x + 1) - \sin(x+1)}{\delta x}$$

⑤

$$\frac{dy}{dx} = \frac{2\cos\left(x + \delta x/2 + 1\right)}{2 \times} \frac{\sin \delta x/2}{\delta x/2} \quad ⑤$$

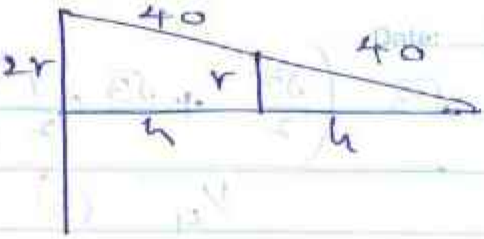
$$\delta x/2 \to 0$$

$$= \cos(x+1) \times 1$$

$$\frac{dy}{dx} = \cos(x+1) \quad ⑤$$

20 / 20

c/



ছিন্নকের আয়তন $= \frac{1}{3}\pi(2r)^2 \times 2h - \frac{1}{3}\pi r^2 h$

$$= \frac{7}{3}\pi r^2 h$$

$$V = \frac{7}{3}\pi r^2 \cdot \sqrt{1600 - r^2}$$

$$\frac{dv}{dr} = \frac{7}{3}\pi\left[\sqrt{1600-r^2} \times 2r + \frac{r^2(-2r)}{2\sqrt{1600-r^2}}\right]$$

~~$0 < r$~~

$$= \frac{7}{3}\pi\left[\frac{(1600-r^2)4r \mp 2r^3}{2\sqrt{1600-r^2}}\right]$$

$$= \frac{7}{3}\pi \frac{1600 \times \overset{2}{\cancel{4r}} - \overset{3}{\cancel{6}}r^3}{\underset{1}{\cancel{2}}\sqrt{1600-r^2}}$$

$$0 < \frac{7\pi}{3}\frac{r(3200-3r^2)}{\sqrt{1600-r^2}}$$

$r = 0$ $\quad r = \frac{3200}{3}$

$0 < r < \frac{3200}{3}$ $\quad r > \frac{3200}{3}$

$\frac{dv}{dr}$ $\quad +$ $\quad -$

$\frac{60}{60}$

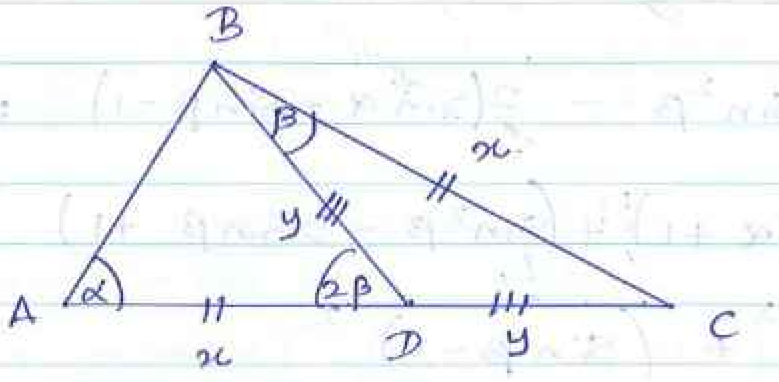
$r = \frac{3200}{3}$ এ সর্বোচ্চ

16/

a, අසත්‍ය වේ.

30

~~b/~~



ABD Δ ට sin සූත්‍රයෙන්

$$\frac{y}{\sin\alpha} = \frac{x}{\sin[\pi-(\alpha+2\beta)]} \Rightarrow \frac{y}{x} = \frac{\sin\alpha}{\sin(\alpha+2\beta)} \quad \text{——①}$$

BDC Δයට sin සූත්‍රයෙන්

$$\frac{y}{\sin\beta} = \frac{x}{\sin(\pi-2\beta)} \Rightarrow \frac{y}{x} = \frac{\sin\beta}{\sin 2\beta} \quad \text{——②}$$

① හා ②න්

$$\frac{\sin\alpha}{\sin(\alpha+2\beta)} = \frac{\sin\beta}{\sin 2\beta}$$

$$\frac{\sin\alpha}{\sin(\alpha+2\beta)} = \frac{\sin\beta}{2\sin\beta\cos\beta}$$

$$2\sin\alpha\cos\beta = \sin(\alpha+2\beta)$$

$\alpha = \beta$

$$2\sin\alpha.\cos\alpha = \sin(\alpha+2\alpha)$$

$$\sin 2\alpha = \sin 3\alpha$$

$$\sin 2\alpha = \sin(\pi-3\alpha)$$

$2\alpha = \pi - 3\alpha$

$\alpha = \pi/5$

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b. $\sin^2\alpha + \sin^2\beta - 2(\sin\alpha + \sin\beta - 1)$ සලකමු

$(\sin^2\alpha - 2\sin\alpha + 1) + (\sin^2\beta - 2\sin\beta + 1)$

$= (\sin\alpha - 1)^2 + (\sin\beta - 1)^2$

$\therefore (\sin\alpha - 1)^2 + (\sin\beta - 1)^2 > 0$

$\therefore \sin^2\alpha + \sin^2\beta - 2(\sin\alpha + \sin\beta - 1) > 0$

$\sin^2\alpha + \sin^2\beta > 2(\sin\alpha + \sin\beta - 1)$

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c. $(\cos x + \cos y)^2 + (\sin x + \sin y)^2$

$= \left[2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]^2 + \left[2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]^2$

$= 4\cos^2\left(\frac{x+y}{2}\right)\left[\cos^2\left(\frac{x+y}{2}\right) + \sin^2\left(\frac{x+y}{2}\right)\right]$

$= 4\cos^2\left(\frac{x-y}{2}\right)$

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17/

L.H.S.

a/. $\operatorname{cosec} x(\sec x - 1) - \cot x(1 - \cos x)$

$\frac{1}{\sin x}\left(\frac{1}{\cos x} - 1\right) - \frac{\cos x}{\sin x}(1 - \cos x)$

$\frac{1 - \cos x}{\sin x \times \cos x} - \frac{\cos x(1 - \cos x)}{\sin x}$

$\frac{(1 - \cos x)}{\sin x}\left(\frac{1}{\cos x} - \cos x\right)$

$= \frac{(1 - \cos x)}{\sin x}\frac{(1 - \cos^2 x)}{\cos x}$

$= \frac{(1 - \cos x)}{\cancel{\sin x}}\frac{\sin^{\cancel{2}} x}{\cos x.}$

$= \frac{(1 - \cos x)}{\cos x.}\sin x.$

$= \tan x - \sin x.$

$= R.H.S.$

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b/. $\underbrace{\tan^{-1}\frac{3}{4}}_{\alpha} + \underbrace{\tan^{-1}\frac{3}{5}}_{\beta} - \underbrace{\tan^{-1}\frac{8}{19}}_{\gamma} = \frac{\pi}{4}$

~~$\alpha + \beta$~~. $\alpha = \tan^{-1} 3/4 \quad \beta = \tan^{-1} 3/5 \quad \gamma = \tan^{-1} 8/19$

$\tan\alpha = 3/4 \quad \tan\beta = 3/5 \quad \tan\gamma = 8/19$

$\alpha + \beta - \gamma \quad \pi/4$

$\alpha + \beta = \pi/4 + \gamma$

$\tan(\alpha+\beta) = \tan(\pi/4 + \gamma)$

$\dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \dfrac{\frac{3}{4} + \frac{3}{5}}{1 - \frac{3}{4} \times \frac{3}{5}} = \dfrac{27}{11}$.

$\tan(\pi/4 + \gamma) = \dfrac{\tan\pi/4 + \tan\gamma}{1 - \tan\pi/4\tan\gamma}$

$= \dfrac{1 + 8/19}{1 - 1 \times \frac{8}{19}} = \dfrac{27}{11}$

$\therefore \tan^{-1} 3/4 + \tan^{-1} 3/5 - \tan^{-1} 8/19 = \pi/4$ or

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π $\quad \sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x.$

$\alpha \qquad \beta \qquad \gamma$

$\sin^{-1}x = \alpha \qquad \sin^{-1}(1-x) = \beta \qquad \cos^{-1}x = \gamma$

$\sin\alpha = x \qquad \sin\beta = 1 - x \qquad \cos\gamma = x$

$\cos\alpha = \sqrt{1-x^2} \qquad \cos\beta = \sqrt{1-(1-x)^2}$

$\alpha + \beta = \gamma$

$\cos(\alpha+\beta) = \cos\gamma$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta = \cos\gamma$

$\sqrt{1-x^2}\,\sqrt{1-(1-x)^2} - x(1-x) = x$

$$\sqrt{1-x^2}\ \sqrt{1-(1-x)^2} = x + x - x^2$$

$$\sqrt{1-x^2} \times \sqrt{1-(1-x)^2} = 2x - x^2$$

$$(1-x^2)\ [1-(1-x)^2] = (2x-x^2)^2$$

~~$1 - x^2 - (1-x^2)(1-x)^2 =$~~

$$(1-x^2)(2x-x^2) = (2x-x^2)^2$$

$$(2x-x^2)(1-x^2) - (2x-x^2)^2 = 0$$

$$(2x-x^2)\left[1 - x^2 - 2x + x^2\right] = 0$$

$$x(2-x)(1-2x) = 0$$

$x = 0$ $x = 2$ $x = 1/2$

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c/ $\sin\theta - \sqrt{3}\cos\theta + 2 = 0$

$$2\left(\frac{1}{2}\sin\theta - \frac{\sqrt{3}}{2}\cos\theta\right) + 2 = 0$$

$$2\left(\sin\theta\cos\pi/3 - \cos\theta\sin\pi/3\right) + 2 = 0$$

$$2\ \sin(\theta + \pi/3) + 2 = 0$$

$$\sin(\theta - \pi/3) + 1 = 0$$

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$y = \sin\theta - \sqrt{3}\cos\theta + 2$

$y = 2\sin(\theta - \pi/3) + 2$

$-1 \le \sin(\theta - \pi/3) \le 1$

$-2 \le 2\sin(\theta - \pi/3) \le 2$

$-2 + 2 \le 2\sin(\theta - \pi/3) + 2 \le 2 + 2$

$0 \le y \le 4$

$y = 2\sin(\theta - \pi/3) + 2 \quad (-\pi/2 \le \theta < \pi/2)$

$0 \le y \le 4$

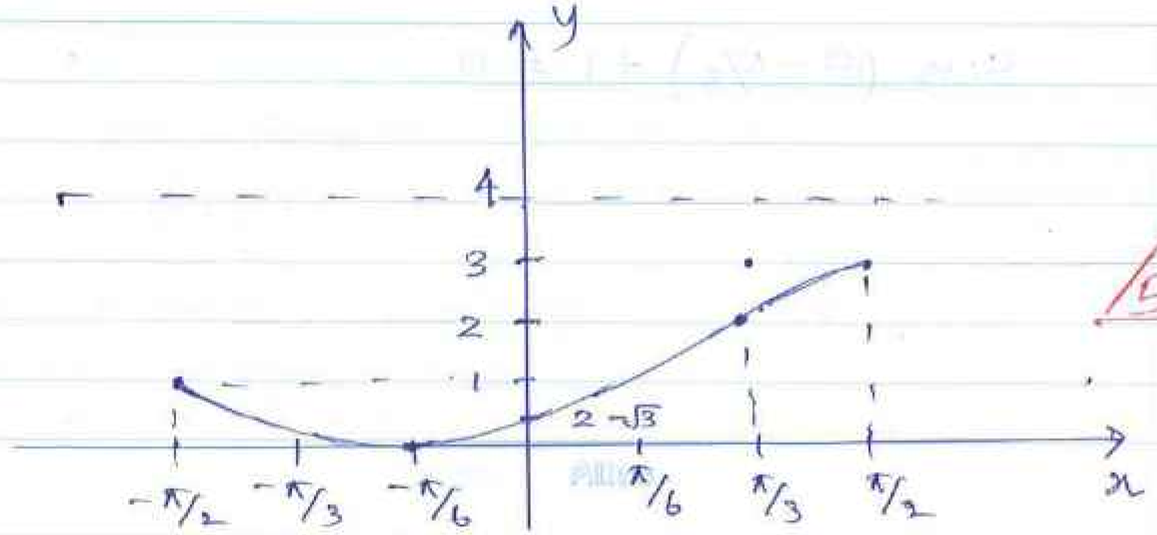
$\theta = \pi/2$ so $y = 3$

$\theta = 0 \quad y = 2 - \sqrt{3}$

$\theta = -\pi/2 \quad y = 1$

$\theta = \pi/3 \quad y = 2$

$\theta = -\pi/6 \quad y = 0$



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